

Introductory
STATISTICS

9TH EDITION



Neil
WEISS

Chapter 6

The Normal Distribution



Section 6.1

Introducing Normally Distributed Variables



Figure 6.2

Three normal distributions

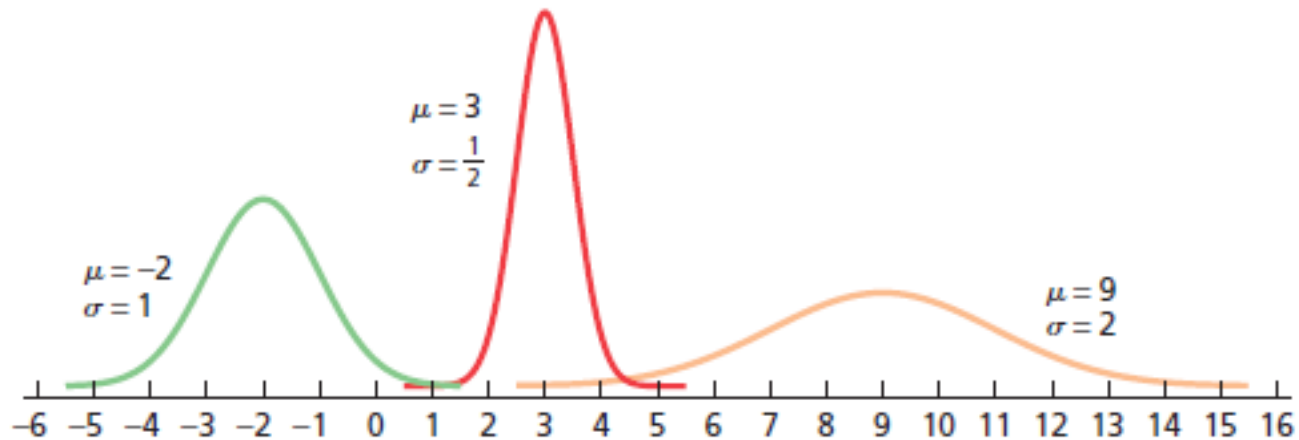


Table 6.1

Frequency and relative-frequency distributions for heights

Height (in.)	Frequency f	Relative frequency
56–under 57	3	0.0009
57–under 58	6	0.0018
58–under 59	26	0.0080
59–under 60	74	0.0227
60–under 61	147	0.0450
61–under 62	247	0.0757
62–under 63	382	0.1170
63–under 64	483	0.1480
64–under 65	559	0.1713
65–under 66	514	0.1575
66–under 67	359	0.1100
67–under 68	240	0.0735
68–under 69	122	0.0374
69–under 70	65	0.0199
70–under 71	24	0.0074
71–under 72	7	0.0021
72–under 73	5	0.0015
73–under 74	1	0.0003
	3264	1.0000

Figure 6.4

Relative-frequency histogram for heights with superimposed normal curve

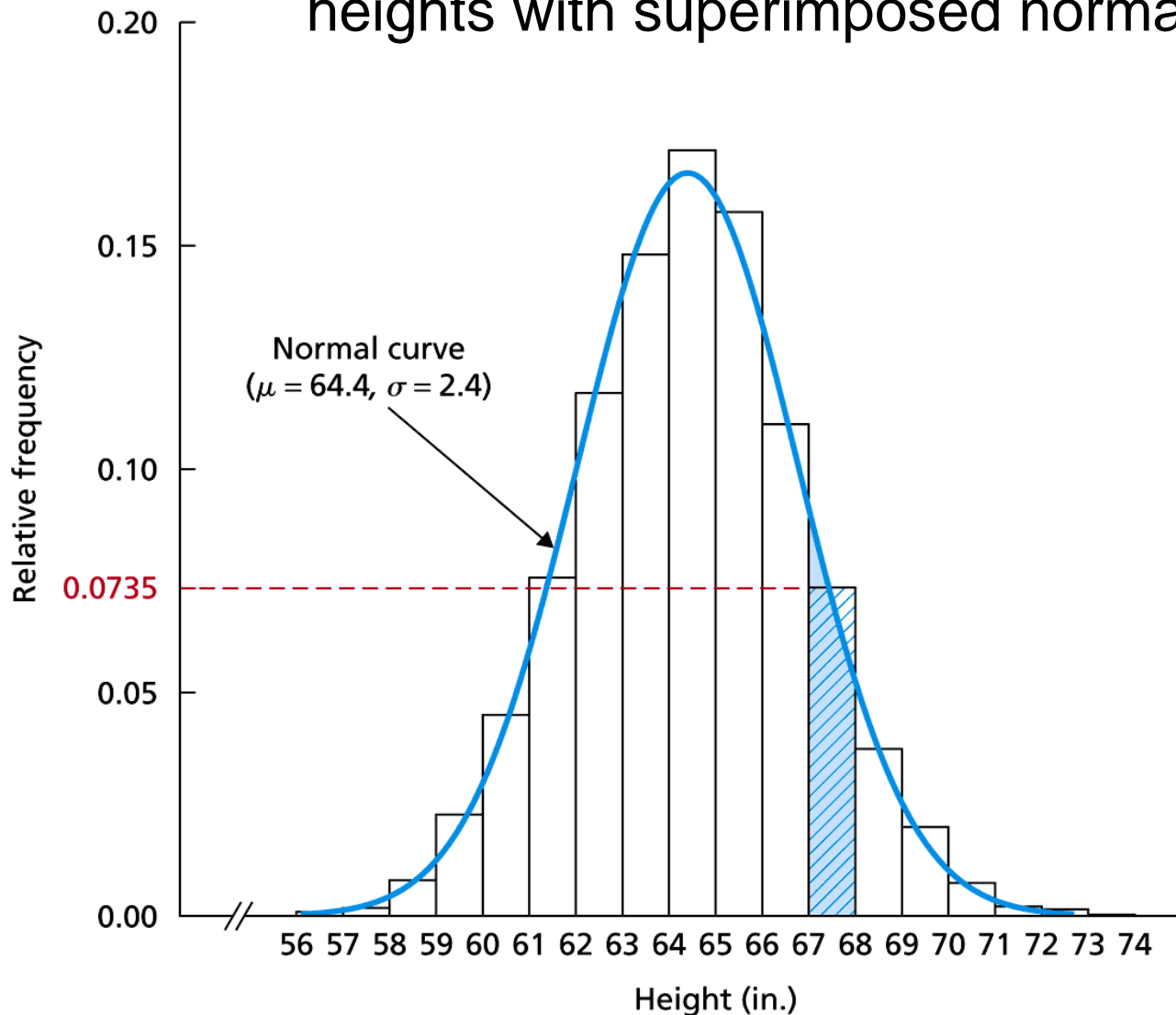


Figure 6.6

Standardizing normal distributions

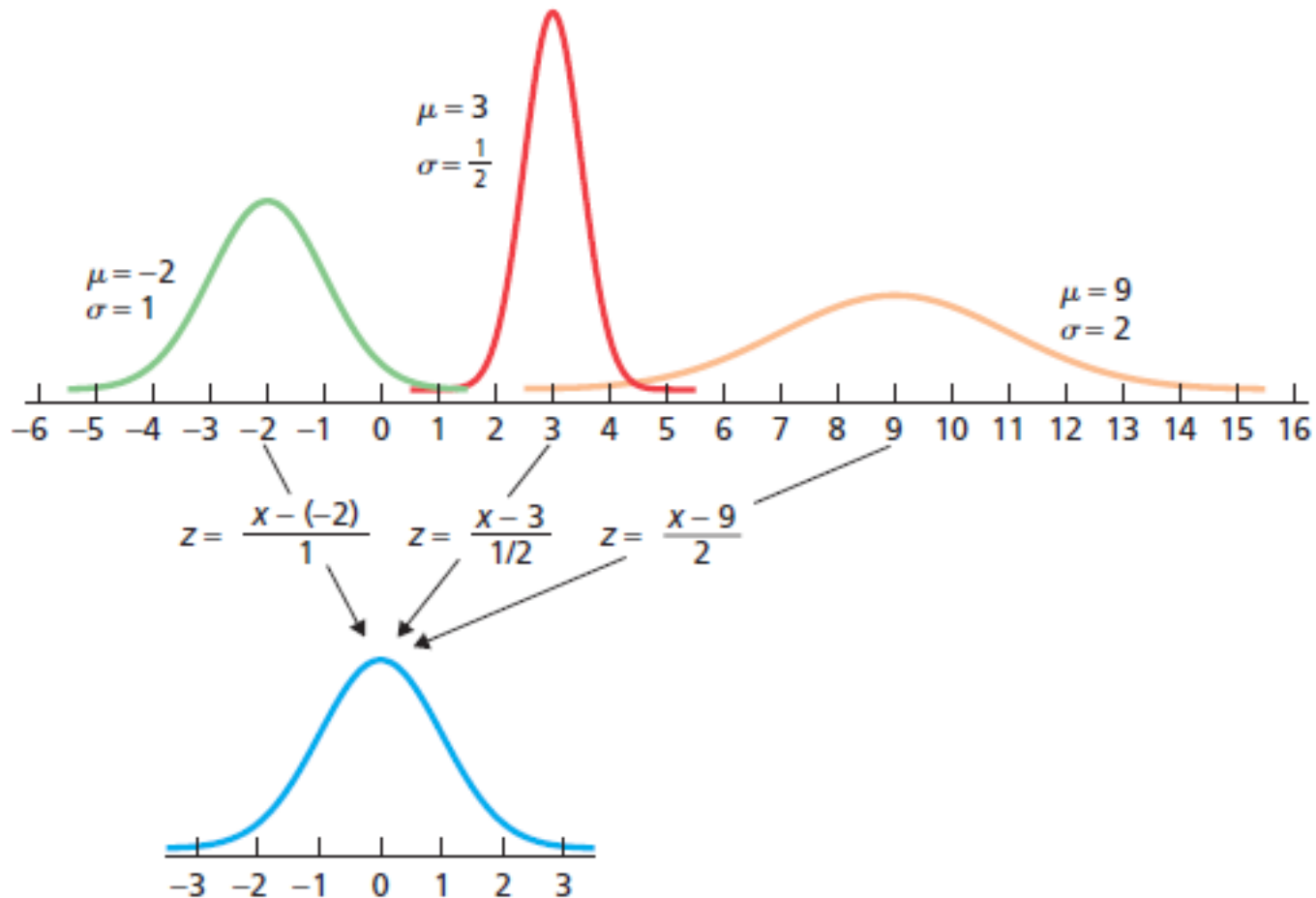
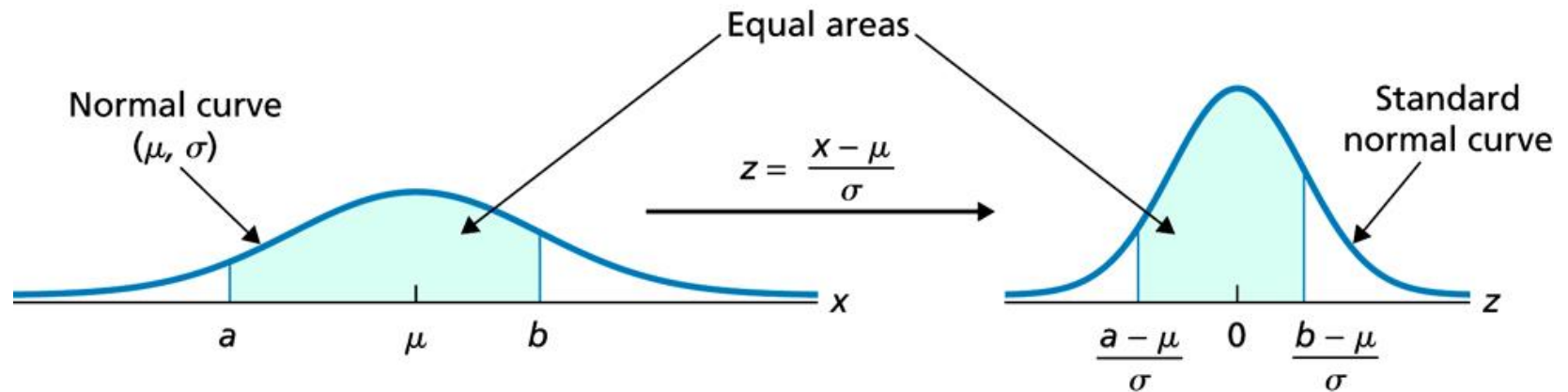


Figure 6.7

Finding percentages for a normally distributed variable from areas under the standard normal curve



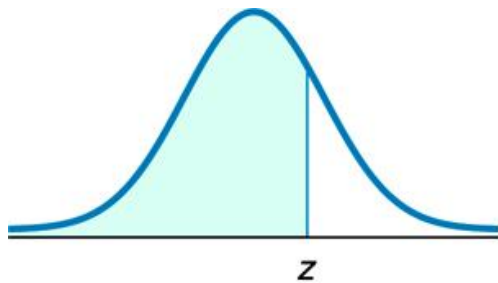
Section 6.2

Areas Under the Standard Normal Curve

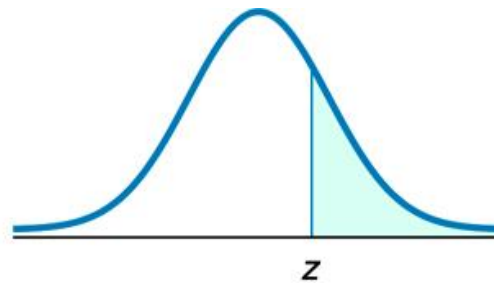


Figure 6.12

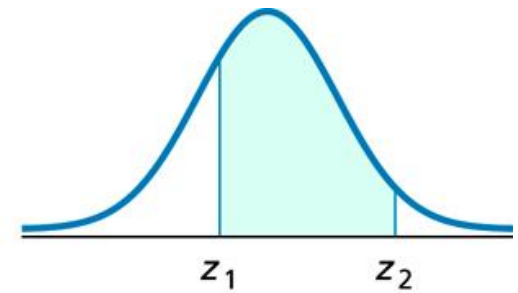
Using Table II to find the area under the standard normal curve that lies (a) to the left of a specified z-score, (b) to the right of a specified z-score, and (c) between two specified z-scores



(a) Shaded area:
Area to left of z



(b) Shaded area:
 $1 - (\text{Area to left of } z)$



(c) Shaded area:
 $(\text{Area to left of } z_2) - (\text{Area to left of } z_1)$

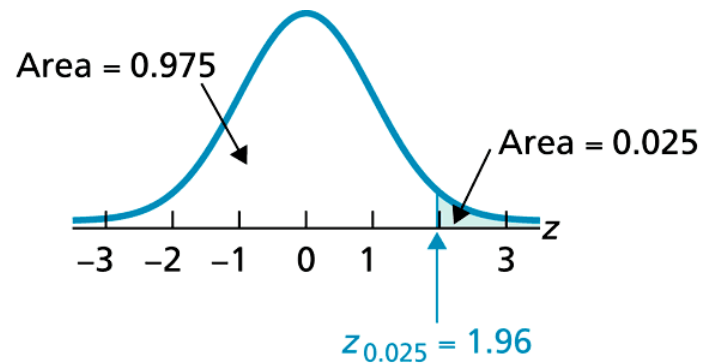
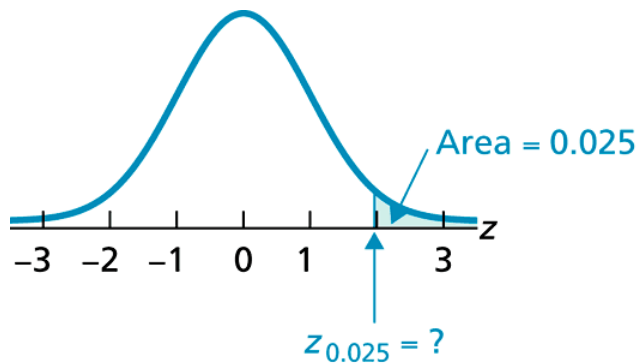
Table 6.2

Areas under the standard normal curve

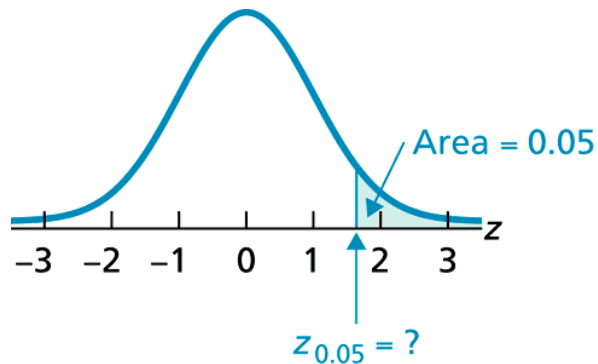
Second decimal place in z										
<i>0.09</i>	<i>0.08</i>	<i>0.07</i>	<i>0.06</i>	<i>0.05</i>	<i>0.04</i>	<i>0.03</i>	<i>0.02</i>	<i>0.01</i>	<i>0.00</i>	z
.
.
.
0.0233	0.0239	0.0244	0.0250	0.0256	0.0262	0.0268	0.0274	0.0281	0.0287	-1.9
0.0294	0.0301	0.0307	0.0314	0.0322	0.0329	0.0336	0.0344	0.0351	0.0359	-1.8
0.0367	0.0375	0.0384	0.0392	0.0401	0.0409	0.0418	0.0427	0.0436	0.0446	-1.7
0.0455	0.0465	0.0475	0.0485	0.0495	0.0505	0.0516	0.0526	0.0537	0.0548	-1.6
0.0559	0.0571	0.0582	0.0594	0.0606	0.0618	0.0630	0.0643	0.0655	0.0668	-1.5
.
.
.

Figures 6.15 & 6.16

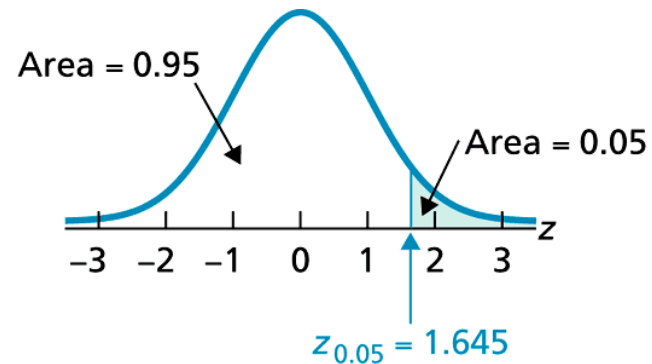
Finding $z_{0.025}$



Finding $z_{0.05}$



(a)



(b)

Section 6.3

Working with Normally Distributed Variables



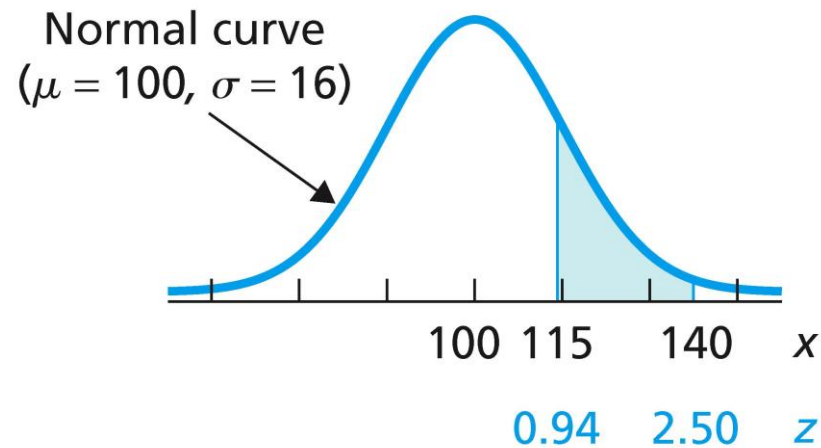
Procedure 6.1

To Determine a Percentage or Probability for a Normally Distributed Variable

- Step 1** Sketch the normal curve associated with the variable.
- Step 2** Shade the region of interest and mark its delimiting x -value(s).
- Step 3** Find the z -score(s) for the delimiting x -value(s) found in Step 2.
- Step 4** Use Table II to find the area under the standard normal curve delimited by the z -score(s) found in Step 3.

Figure 6.19

Determination of the percentage of people having IQs between 115 and 140



Key Fact 6.6 & Figure 6.20

The 68.26-95.44-99.74 Rule

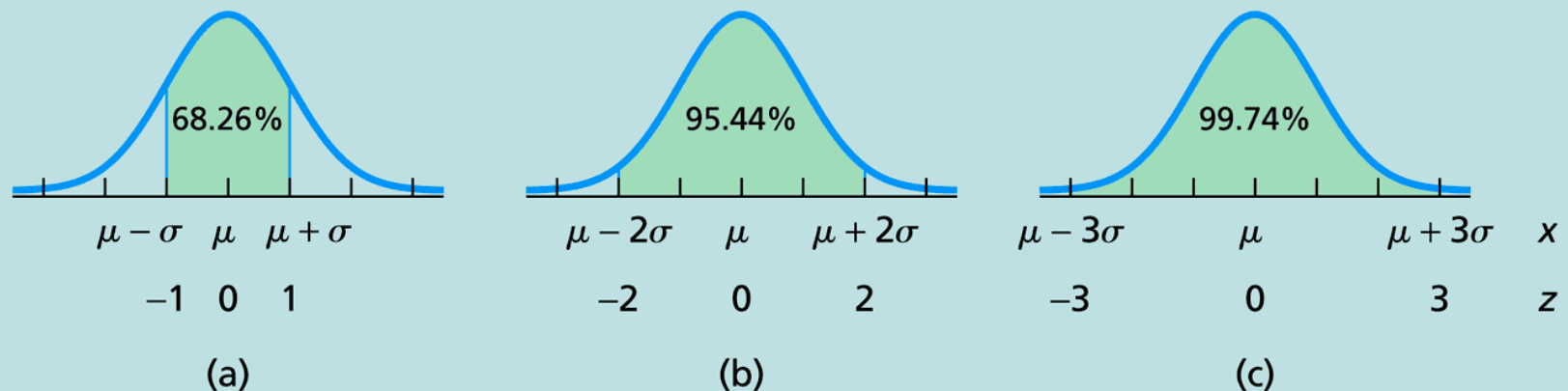
Any normally distributed variable has the following properties.

Property 1: 68.26% of all possible observations lie within one standard deviation to either side of the mean, that is, between $\mu - \sigma$ and $\mu + \sigma$.

Property 2: 95.44% of all possible observations lie within two standard deviations to either side of the mean, that is, between $\mu - 2\sigma$ and $\mu + 2\sigma$.

Property 3: 99.74% of all possible observations lie within three standard deviations to either side of the mean, that is, between $\mu - 3\sigma$ and $\mu + 3\sigma$.

These properties are illustrated in Fig. 6.20.



Procedure 6.2

To Determine the Observations Corresponding to a Specified Percentage or Probability for a Normally Distributed Variable

Step 1 Sketch the normal curve associated with the variable.

Step 2 Shade the region of interest.

Step 3 Use Table II to determine the z -score(s) delimiting the region found in Step 2.

Step 4 Find the x -value(s) having the z -score(s) found in Step 3.

Section 6.4

Assessing Normality; Normal Probability Plots



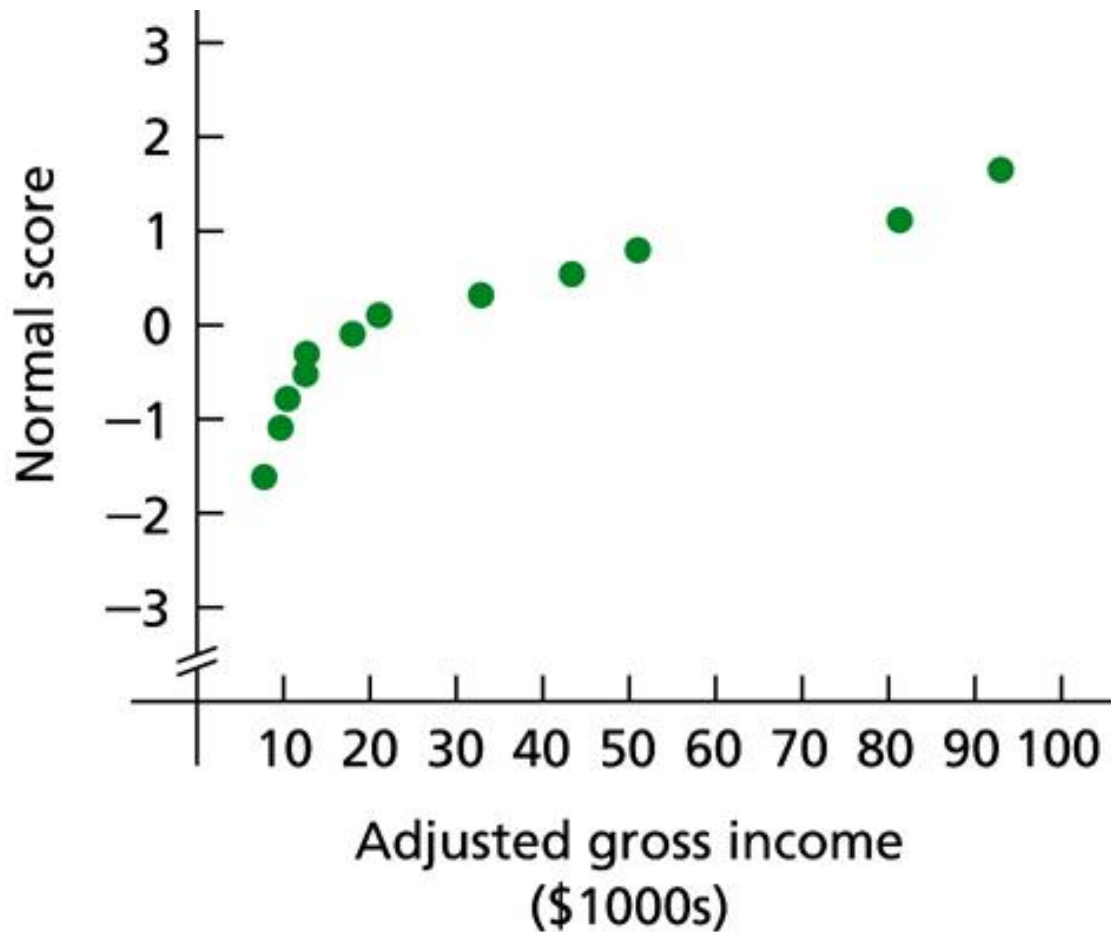
Table 6.4

Ordered data and normal scores

Adjusted gross income	Normal score
7.8	-1.64
9.7	-1.11
10.6	-0.79
12.7	-0.53
12.8	-0.31
18.1	-0.10
21.2	0.10
33.0	0.31
43.5	0.53
51.1	0.79
81.4	1.11
93.1	1.64

Figure 6.23

Normal probability plot for the sample of adjusted gross incomes



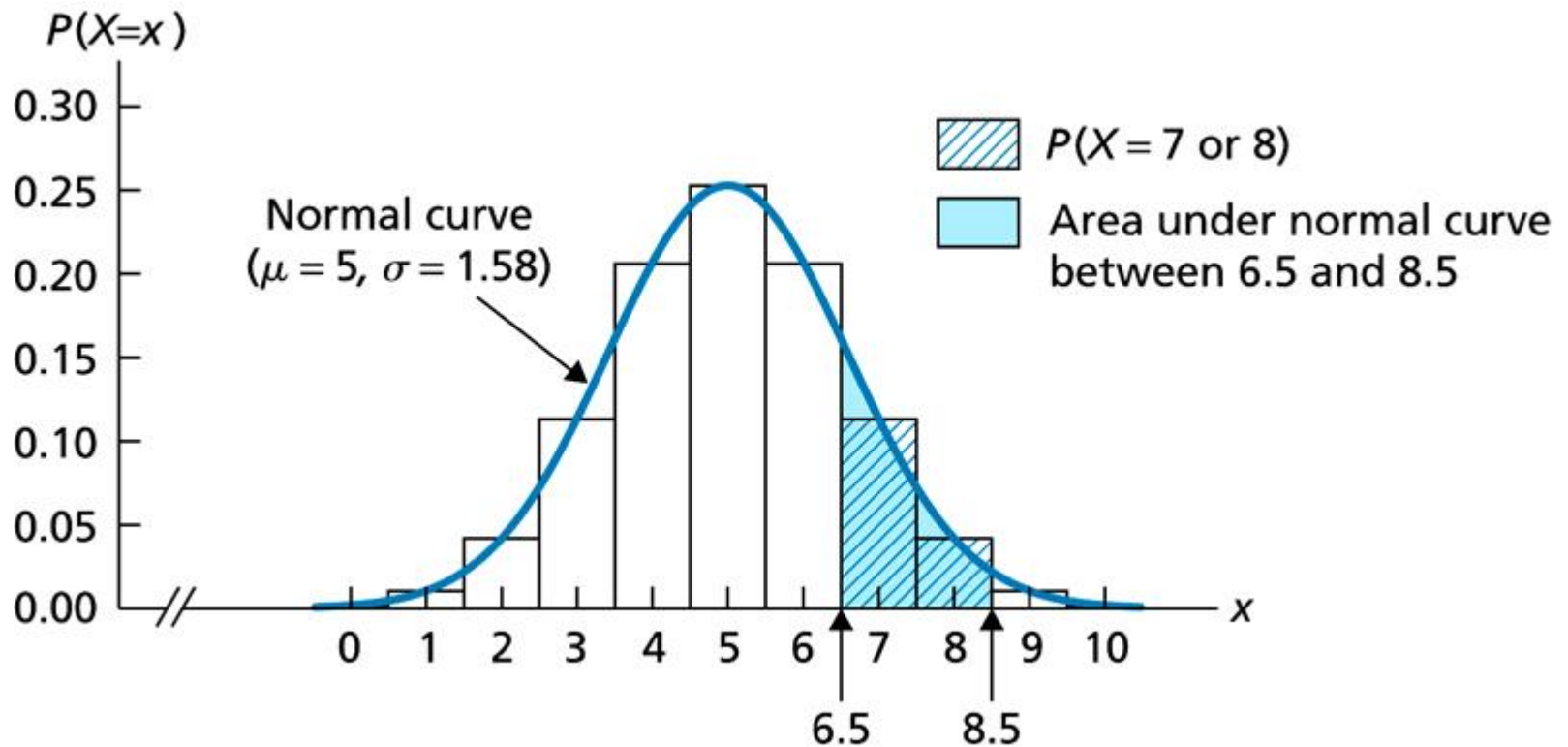
Section 6.5

Normal Approximation to the Binomial Distribution



Figure 6.25

Probability histogram for X with superimposed normal curve



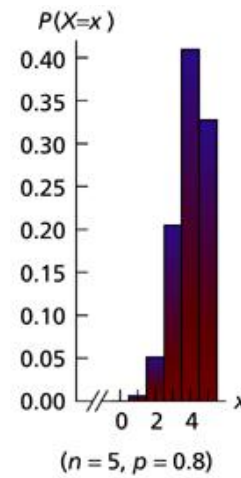
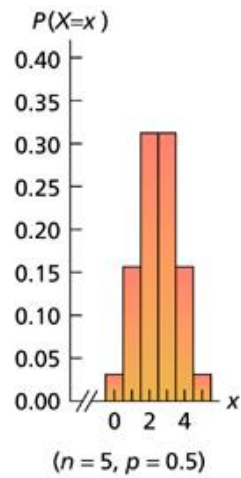
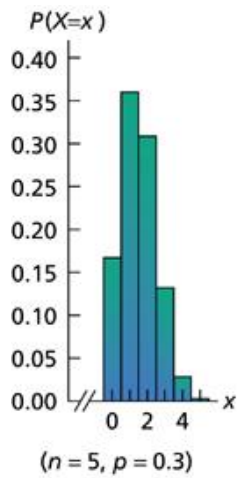
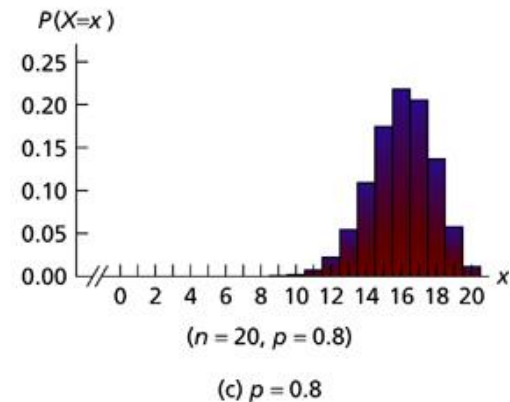
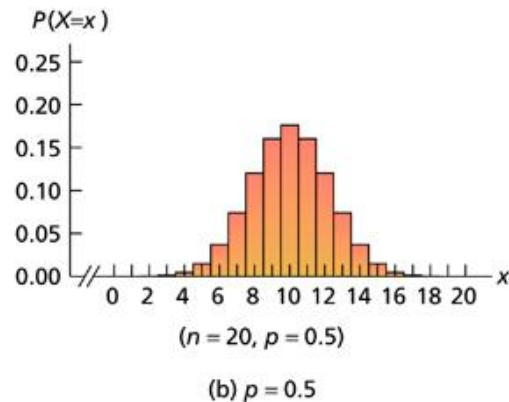
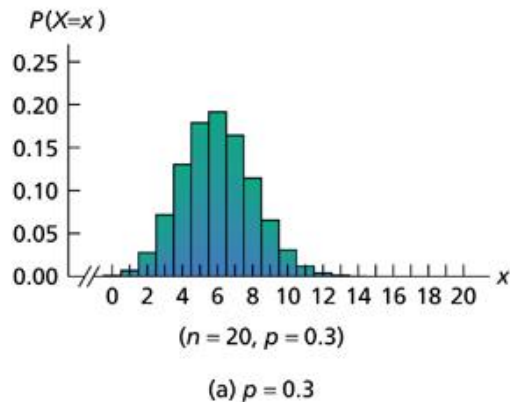
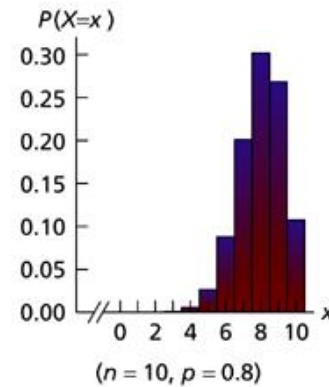
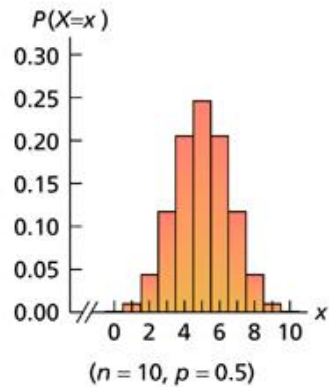
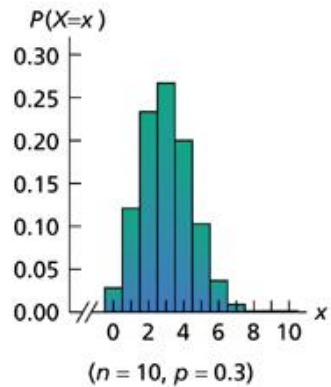


Figure 6.27
Nine different
binomial
distributions



Procedure 6.3

To Approximate Binomial Probabilities by Normal-Curve Areas

Step 1 Find n , the number of trials, and p , the success probability.

Step 2 Continue only if both np and $n(1 - p)$ are 5 or greater.

Step 3 Find μ and σ , using the formulas $\mu = np$ and $\sigma = \sqrt{np(1 - p)}$.

Step 4 Make the correction for continuity, and find the required area under the normal curve with parameters μ and σ .