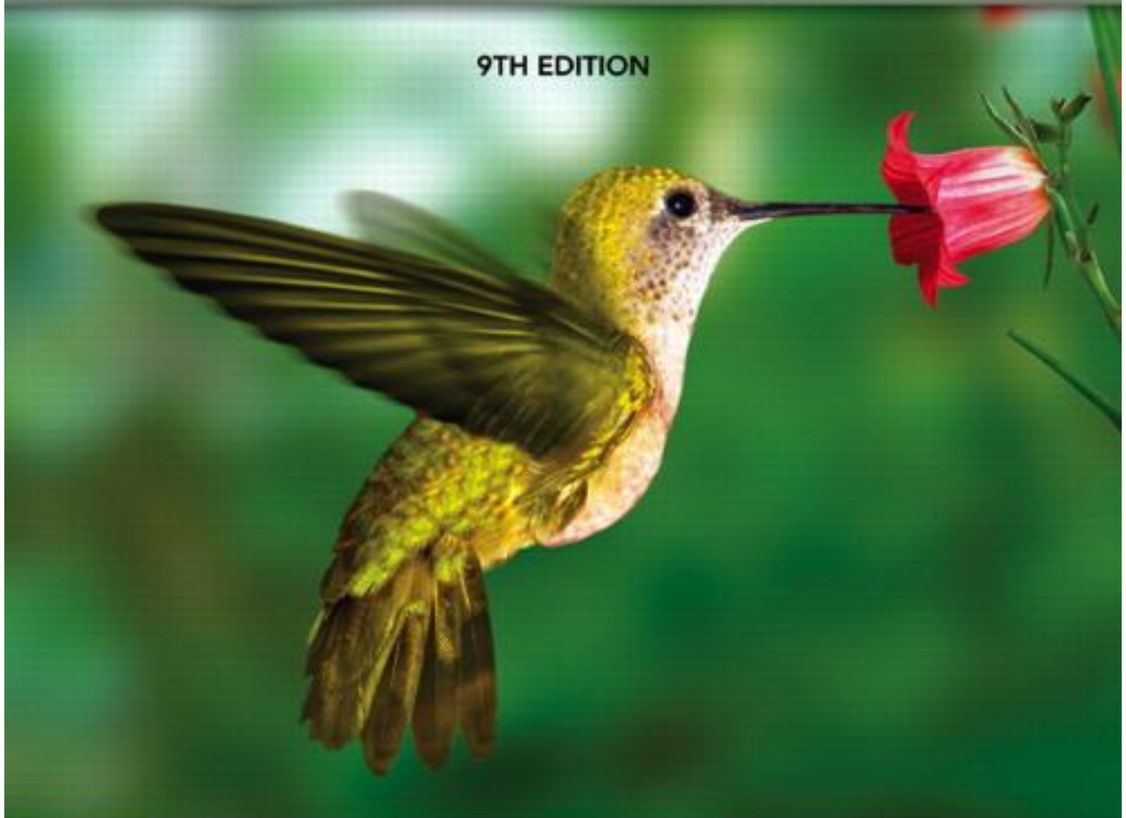
Introductory STATISTICS



9TH EDITION

Neil WEISS

Chapter 8

Confidence Intervals for One Population Mean

Section 8.1

Estimating a Population Mean

Definition 8.1

Point Estimate

A **point estimate** of a parameter is the value of a statistic used to estimate the parameter.

Definition 8.2

Confidence-Interval Estimate

Confidence interval (CI): An interval of numbers obtained from a point estimate of a parameter.

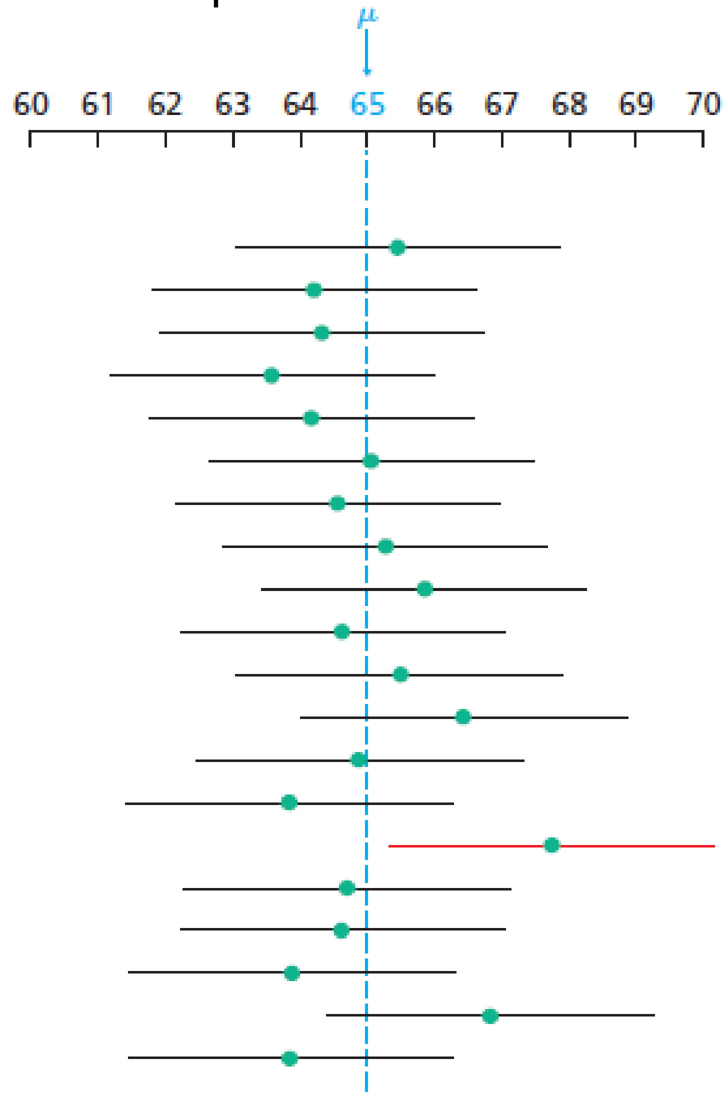
Confidence level: The confidence we have that the parameter lies in the confidence interval (i.e., that the confidence interval contains the parameter).

Confidence-interval estimate: The confidence level and confidence interval.

Figure 8.2

Twenty confidence intervals for the mean price of all new mobile homes, each based on a sample of 36 new mobile homes

Sample	$\bar{x}$	95.44% CI	μ in CI?
1	65.45	63.06 to 67.85	yes
2	64.21	61.81 to 66.61	yes
3	64.33	61.93 to 66.73	yes
4	63.59	61.19 to 65.99	yes
5	64.17	61.77 to 66.57	yes
6	65.07	62.67 to 67.47	yes
7	64.56	62.16 to 66.96	yes
8	65.28	62.88 to 67.68	yes
9	65.87	63.48 to 68.27	yes
10	64.61	62.22 to 67.01	yes
11	65.51	63.11 to 67.91	yes
12	66.45	64.05 to 68.85	yes
13	64.88	62.48 to 67.28	yes
14	63.85	61.45 to 66.25	yes
15	67.73	65.33 to 70.13	no
16	64.70	62.30 to 67.10	yes
17	64.60	62.20 to 67.00	yes
18	63.88	61.48 to 66.28	yes
19	66.82	64.42 to 69.22	yes
20	63.84	61.45 to 66.24	yes



Section 8.2

Confidence Intervals for One Population Mean when Sigma Is Known

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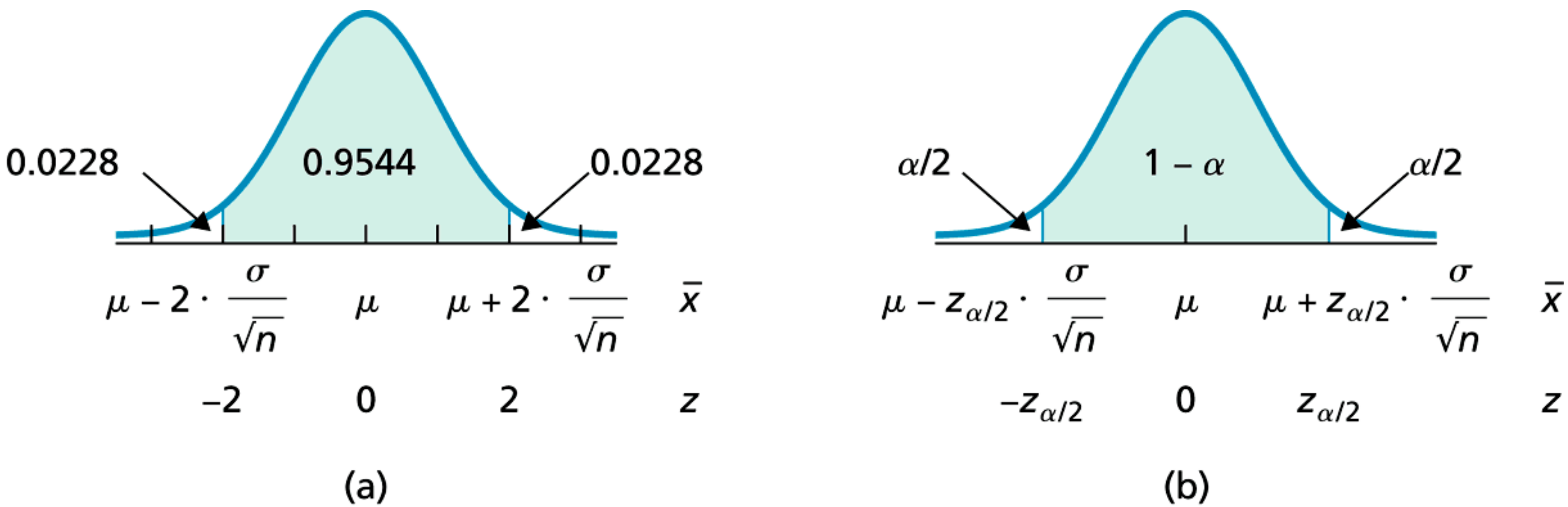
Figure 8.3

(a) 95.44% of all samples have means within 2 standard deviations of μ; (b) 100(1 − α)% of all samples have means within $z_{\alpha/2}$ standard deviations of μ



Procedure 8.1

One-Mean z-Interval Procedure

Purpose To find a confidence interval for a population mean, μ

Assumptions

1. Simple random sample
2. Normal population or large sample
3. σ known

Step 1 **For a confidence level of $1 - \alpha$, use Table II to find $z_{\alpha/2}$.**

Step 2 **The confidence interval for μ is from**

$$\bar{x} - z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \quad \text{to} \quad \bar{x} + z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}},$$

where $z_{\alpha/2}$ is found in Step 1, n is the sample size, and $\bar{x}$ is computed from the sample data.

Step 3 **Interpret the confidence interval.**

Note: The confidence interval is exact for normal populations and is approximately correct for large samples from nonnormal populations.

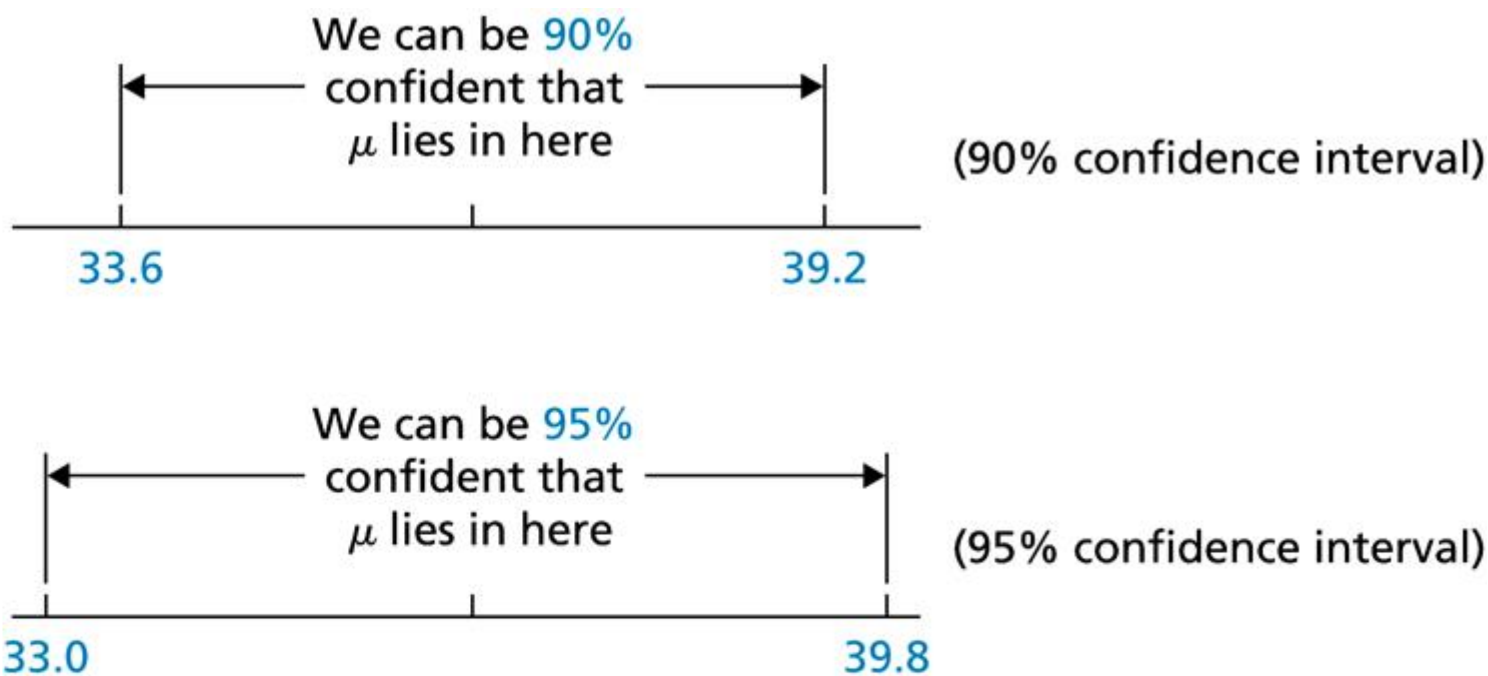
Table 8.3

Ages, in years, of 50 randomly selected people in the civilian labor force

22	58	40	42	43
32	34	45	38	19
33	16	49	29	30
43	37	19	21	62
60	41	28	35	37
51	37	65	57	26
27	31	33	24	34
28	39	43	26	38
42	40	31	34	38
35	29	33	32	33

Figure 8.5

90% and 95% confidence intervals for μ, using the data in Table 8.3



Section 8.3

Margin of Error

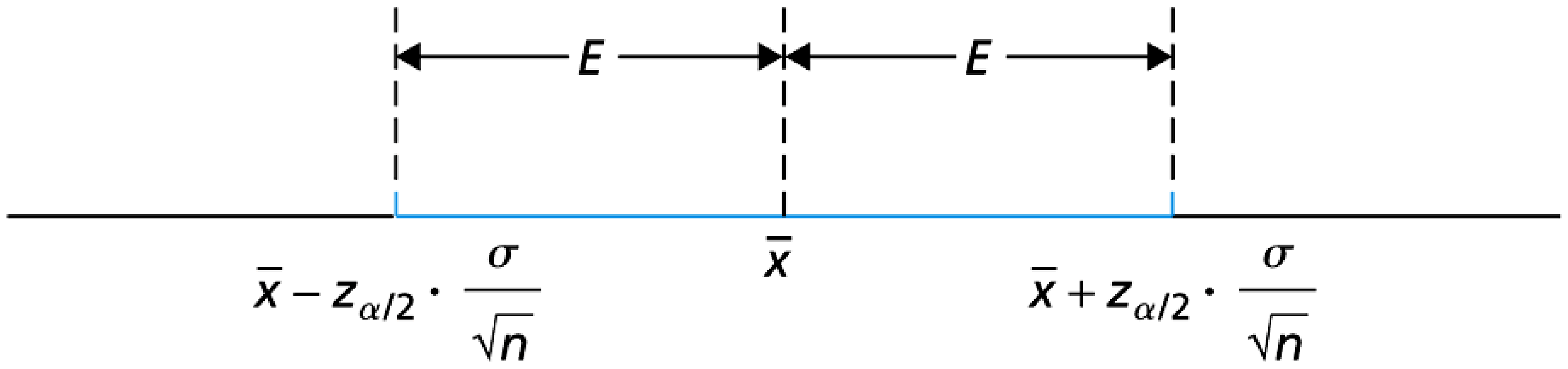
Definition 8.3 & Figure 8.7

Margin of Error for the Estimate of μ

The **margin of error** for the estimate of μ is

$$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}.$$

Figure 8.7 illustrates the margin of error.



Formula 8.1

Sample Size for Estimating μ

The sample size required for a $(1 - \alpha)$-level confidence interval for μ with a specified margin of error, E, is given by the formula

$$n = \left(\frac{z_{\alpha/2} \cdot \sigma}{E} \right)^2,$$

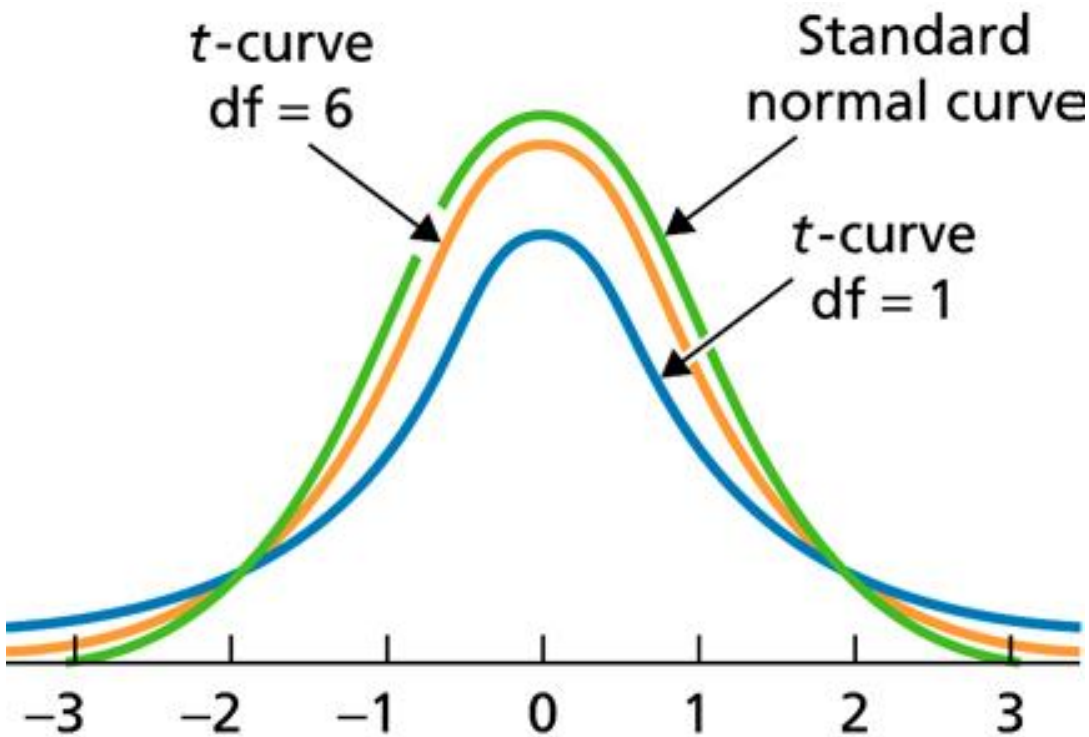
rounded up to the nearest whole number.

Section 8.4

Confidence Intervals for One Population Mean When Sigma Is Unknown

Figure 8.8

Standard normal curve and two t-curves



Key Fact 8.6

Basic Properties of *t*-Curves

Property 1: The total area under a *t*-curve equals 1.

Property 2: A *t*-curve extends indefinitely in both directions, approaching, but never touching, the horizontal axis as it does so.

Property 3: A *t*-curve is symmetric about 0.

Property 4: As the number of degrees of freedom becomes larger, *t*-curves look increasingly like the standard normal curve.

Table 8.4

Values of

df	$t_{0.10}$	$t_{0.05}$	$t_{0.025}$	$t_{0.01}$	$t_{0.005}$	df
⋮	⋮	⋮	⋮	⋮	⋮	⋮
12	1.356	1.782	2.179	2.681	3.055	*12*
13	1.350	1.771	2.160	2.650	3.012	*13*
14	1.345	1.761	2.145	2.624	2.977	*14*
15	1.341	1.753	2.131	2.602	2.947	*15*
⋮	⋮	⋮	⋮	⋮	⋮	⋮

Procedure 8.2

One-Mean *t*-Interval Procedure

Purpose To find a confidence interval for a population mean, μ

Assumptions

1. Simple random sample
2. Normal population or large sample
3. σ unknown

Step 1 **For a confidence level of $1 - \alpha$, use Table IV to find $t_{\alpha/2}$ with df $= n - 1$, where n is the sample size.**

Step 2 **The confidence interval for μ is from**

$$\bar{x} - t_{\alpha/2} \cdot \frac{s}{\sqrt{n}} \quad \text{to} \quad \bar{x} + t_{\alpha/2} \cdot \frac{s}{\sqrt{n}},$$

where $t_{\alpha/2}$ is found in Step 1 and $\bar{x}$ and s are computed from the sample data.

Step 3 **Interpret the confidence interval.**

Note: The confidence interval is exact for normal populations and is approximately correct for large samples from nonnormal populations.

Table 8.5 & Figure 8.10

Losses ($) for a sample of 25 pickpocket offenses

447	207	627	430	883
313	844	253	397	214
217	768	1064	26	587
833	277	805	653	549
649	554	570	223	443

Normal probability plot of the loss data in Table 8.5

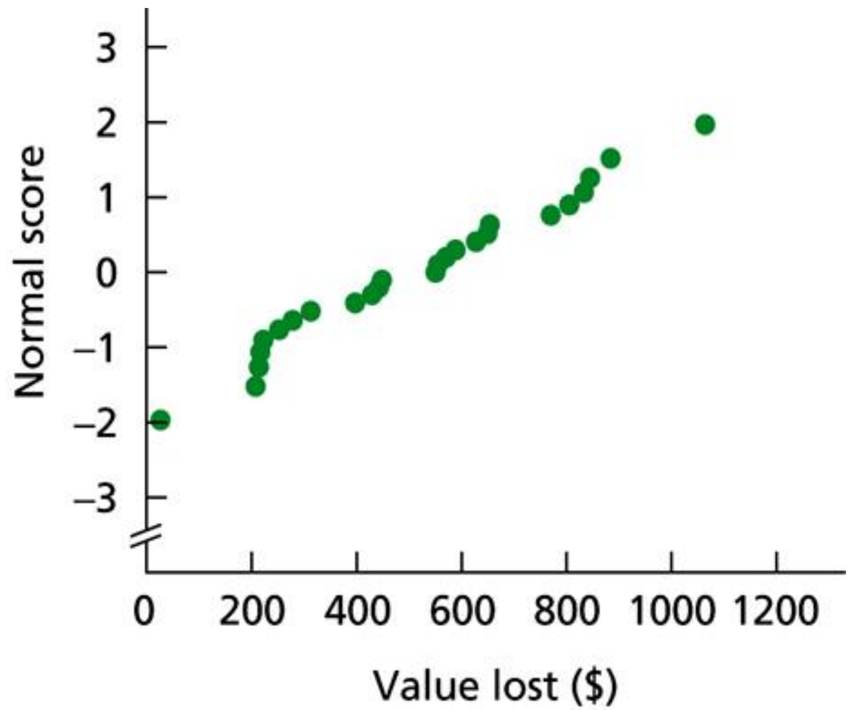


Figure 8.11

Normal probability plots for chicken consumption: (a) original data and (b) data with outlier removed