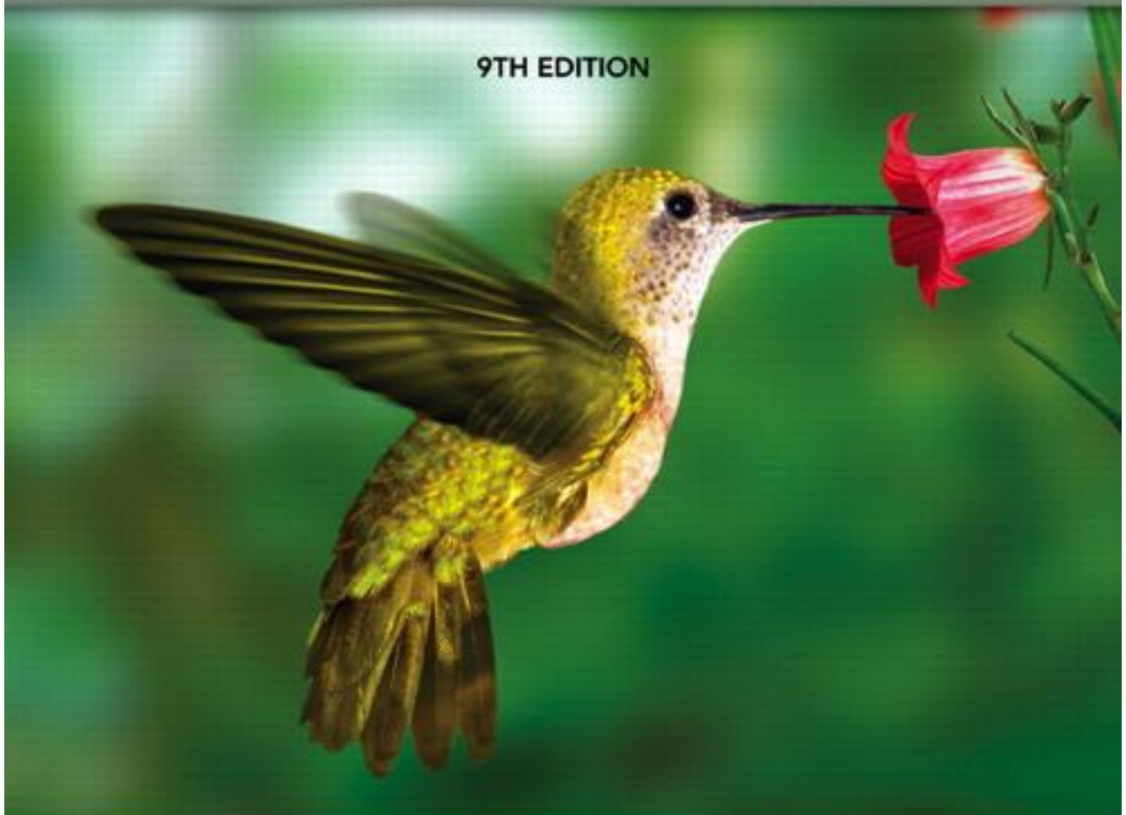
Introductory

STATISTICS

9TH EDITION



Neil
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Chapter 9

Hypothesis Tests for One Population Mean

Section 9.1

The Nature of Hypothesis Testing

Definition 9.1

Null and Alternative Hypotheses; Hypothesis Test

Null hypothesis: A hypothesis to be tested. We use the symbol H_0 to represent the null hypothesis.

Alternative hypothesis: A hypothesis to be considered as an alternative to the null hypothesis. We use the symbol H_a to represent the alternative hypothesis.

Hypothesis test: The problem in a hypothesis test is to decide whether the null hypothesis should be rejected in favor of the alternative hypothesis.

Definition 9.3

Significance Level

The probability of making a Type I error, that is, of rejecting a true null hypothesis, is called the **significance level,** **,** of a hypothesis test.

Section 9.2

Critical-Value Approach to Hypothesis Testing

Figure 9.1

Criterion for deciding whether to reject the null hypothesis

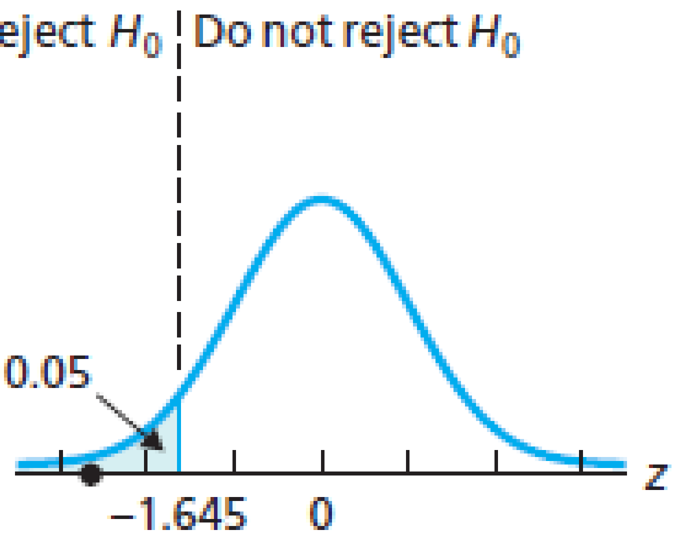


Figure 9.2

Rejection region, nonrejection region, and critical value for the golf-driving-distances hypothesis test

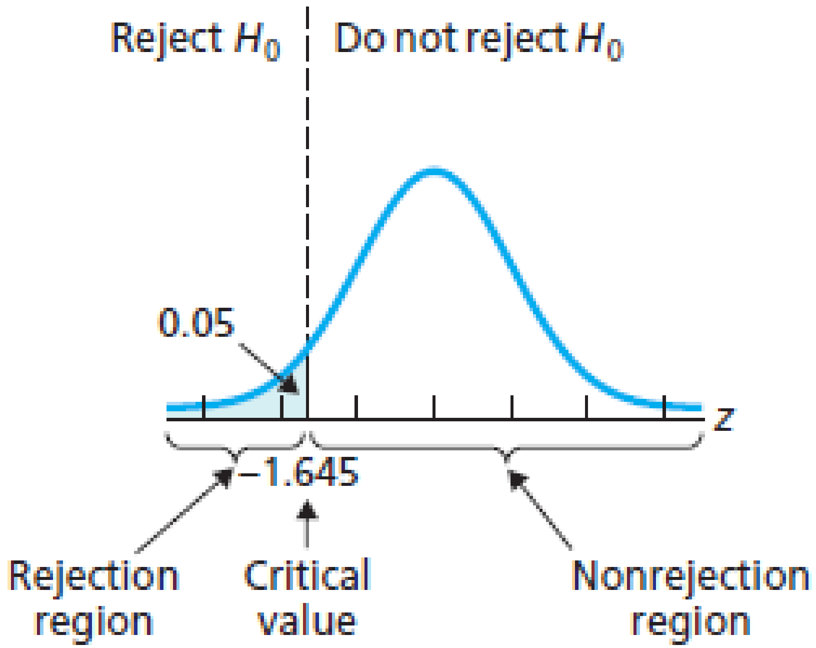


Figure 9.3

Graphical display of rejection regions for two-tailed, left-tailed, and right-tailed tests

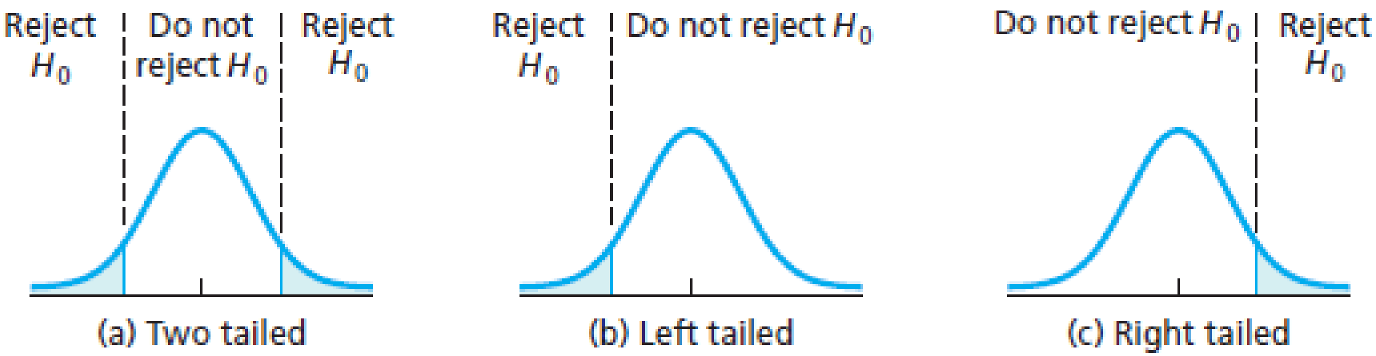
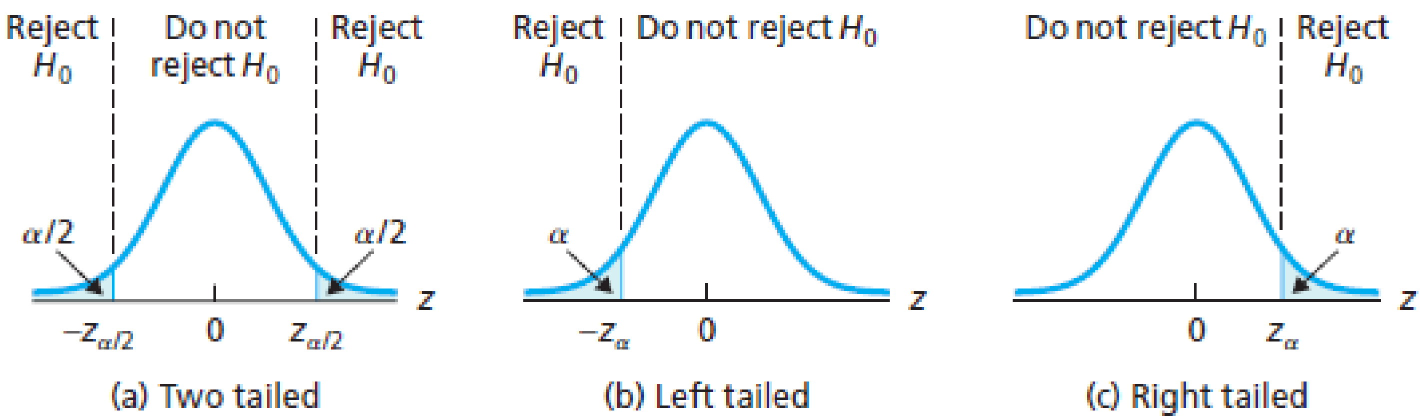


Figure 9.5

Critical value(s) for a hypothesis test at the significance level α if the test is (a) two tailed, (b) left tailed, or (c) right tailed

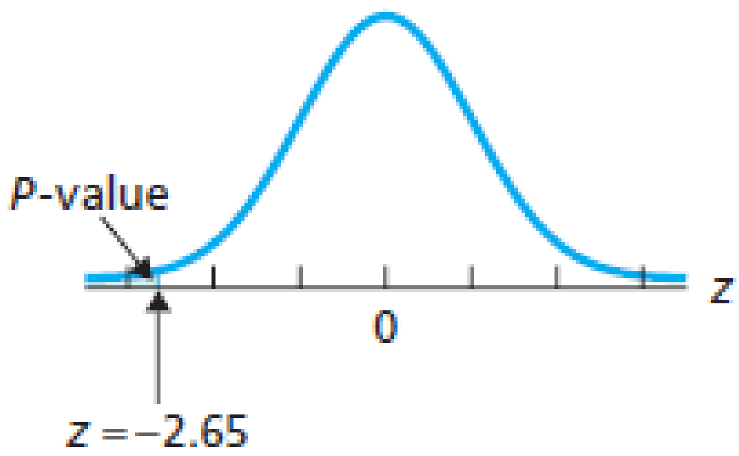


Section 9.3

P-Value Approach to Hypothesis Testing

Figure 9.6

P-value for golf-driving-distances hypothesis test



Definition 9.5

P-Value

The ***P*-value** of a hypothesis test is the probability of getting sample data at least as inconsistent with the null hypothesis (and supportive of the alternative hypothesis) as the sample data actually obtained. We use the letter ***P*** to denote the *P*-value.

Figure 9.7

P-value for a one-mean z-test if the test is (a) two tailed, (b) left tailed, or (c) right tailed

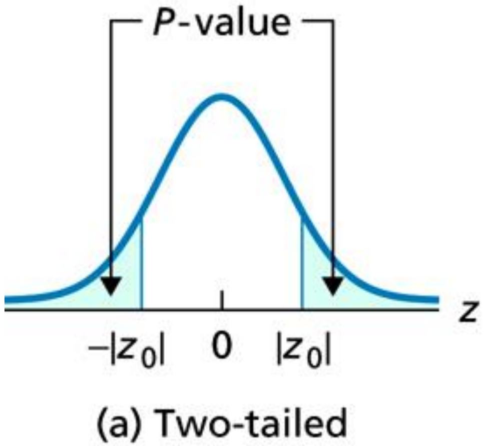


Figure 9.9

Value of the test statistic and the P-value

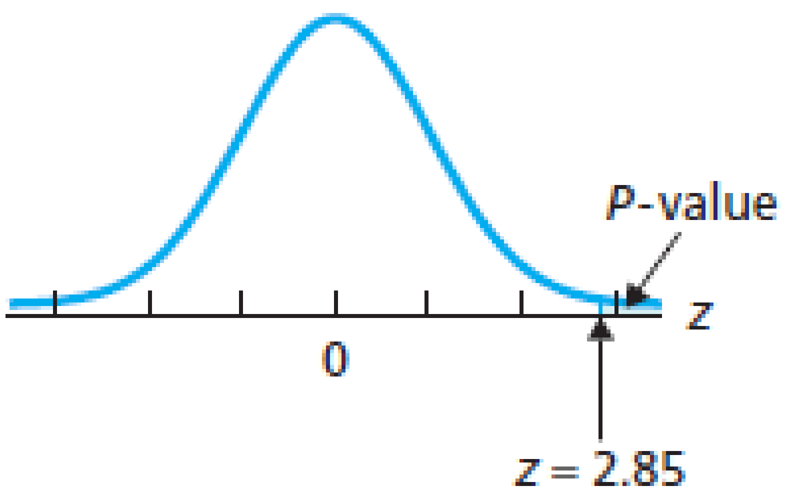


Table 9.8

Guidelines for using the P-value to assess the evidence against the null hypothesis

***P*-value**	**Evidence against H_0**
$P > 0.10$	Weak or none
$0.05 < P \leq 0.10$	Moderate
$0.01 < P \leq 0.05$	Strong
$P \leq 0.01$	Very strong

Section 9.4

Hypothesis Tests for One Population Mean When Sigma Is Known

Procedure 9.1

One-Mean z-Test

Purpose To perform a hypothesis test for a population mean, μ

Assumptions

1. Simple random sample
2. Normal population or large sample
3. σ known

Step 1 **The null hypothesis is H_0: $\mu = \mu_0$, and the alternative hypothesis is**

H_a: $\mu \neq \mu_0$ (Two tailed) **or** H_a: $\mu < \mu_0$ (Left tailed) **or** H_a: $\mu > \mu_0$ (Right tailed)

Step 2 **Decide on the significance level, α.**

Step 3 **Compute the value of the test statistic**

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$$

and denote that value z_0.

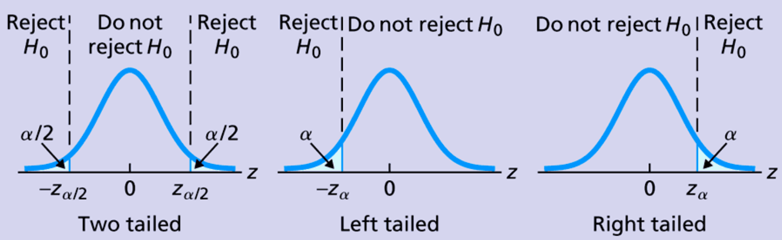
Procedure 9.1 (cont.)

CRITICAL-VALUE APPROACH	OR	P-VALUE APPROACH

Step 4 **The critical value(s) are**

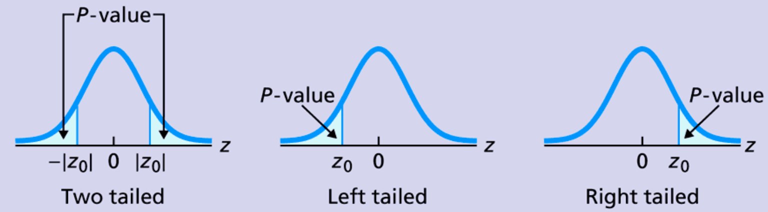
$\pm z_{\alpha/2}$ **(Two tailed)** or $-z_{\alpha}$ **(Left tailed)** or z_{α} **(Right tailed)**

Use Table II to find the critical value(s).



Step 5 **If the value of the test statistic falls in the rejection region, reject H_0; otherwise, do not reject H_0.**

Step 4 **Use Table II to obtain the P-value.**



Step 5 **If $P \leq \alpha$, reject H_0; otherwise, do not reject H_0.**

Step 6 **Interpret the results of the hypothesis test.**

Note: The hypothesis test is exact for normal populations and is approximately correct for large samples from nonnormal populations.

Section 9.5

Hypothesis Tests for One Population Mean When Sigma Is Unknown

Figure 9.17

Estimating the P-value of a left-tailed t-test with a sample size of 12 and test statistic t = −1.938

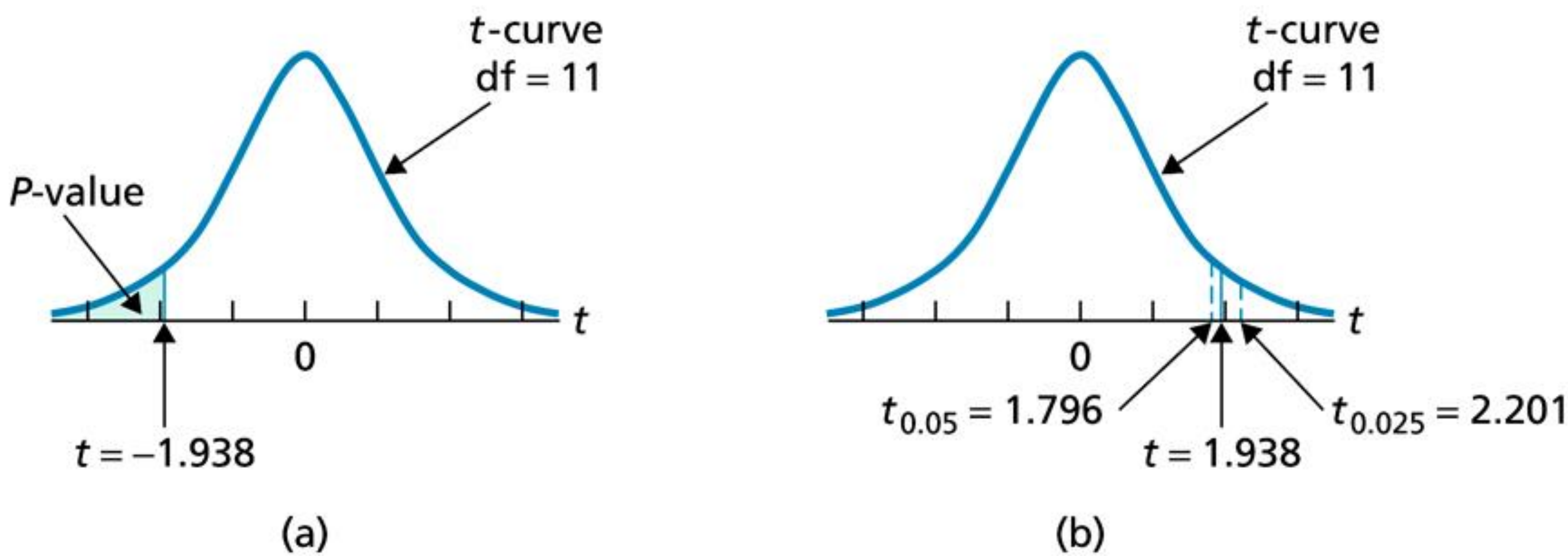
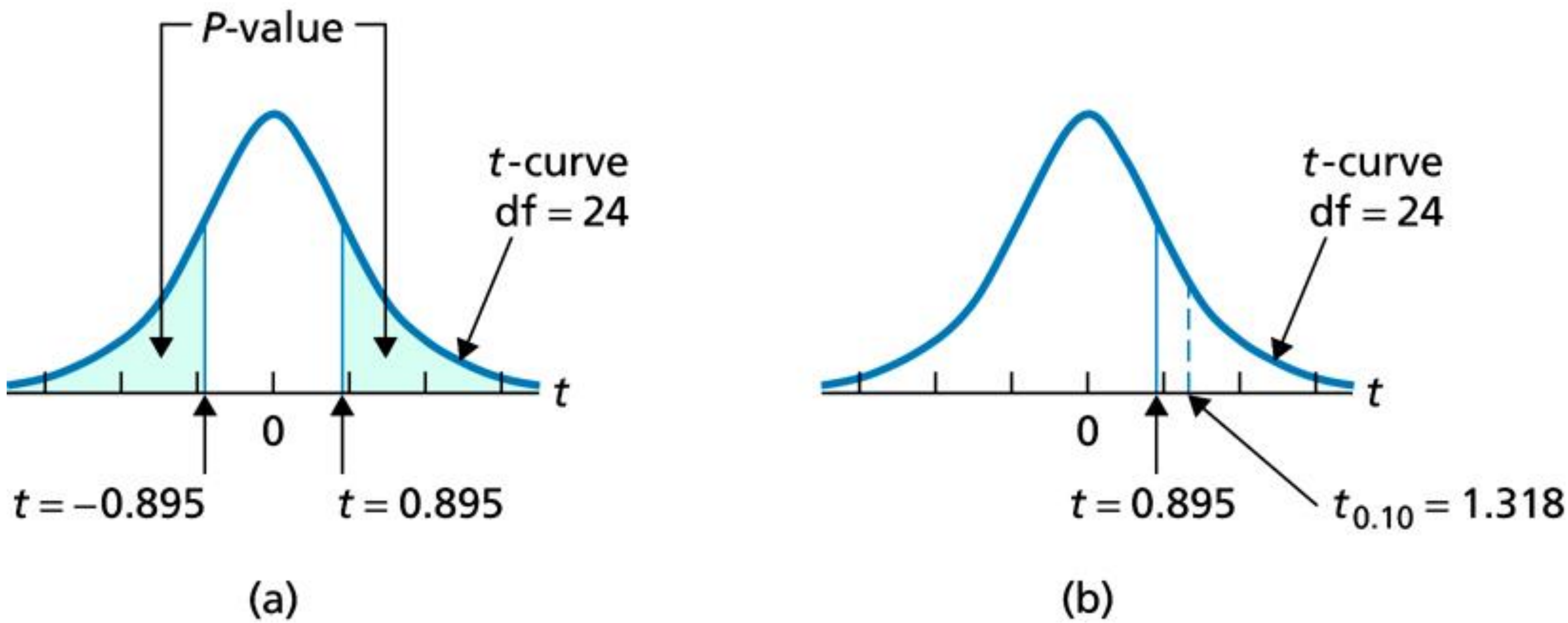


Figure 9.18

Estimating the P-value of a two-tailed t-test with a sample size of 25 and test statistic t =−0.895



Procedure 9.2

One-Mean *t*-Test

Purpose To perform a hypothesis test for a population mean, μ

Assumptions

1. Simple random sample
2. Normal population or large sample
3. σ unknown

Step 1 **The null hypothesis is H_0: $\mu = \mu_0$, and the alternative hypothesis is**

H_a: $\mu \neq \mu_0$	or	H_a: $\mu < \mu_0$	or	H_a: $\mu > \mu_0$
(Two tailed)		**(Left tailed)**		**(Right tailed)**

Step 2 **Decide on the significance level, α.**

Step 3 **Compute the value of the test statistic**

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

and denote that value t_0.

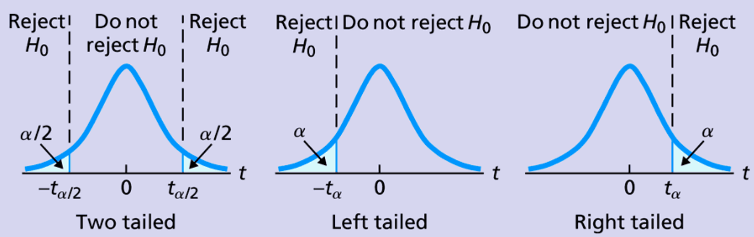
Procedure 9.2 (cont.)

CRITICAL-VALUE APPROACH	OR	*P*-VALUE APPROACH

Step 4 The critical value(s) are

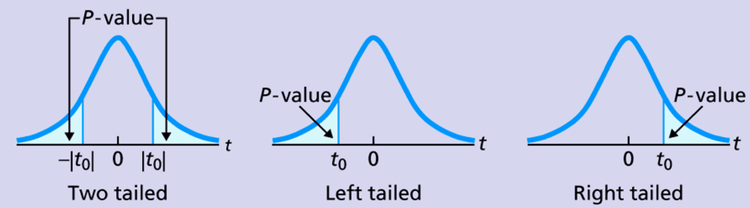
$\pm t_{\alpha/2}$ (Two tailed) or $-t_{\alpha}$ (Left tailed) or t_{α} (Right tailed)

with df $= n - 1$. Use Table IV to find the critical value(s).



Step 5 If the value of the test statistic falls in the rejection region, reject H_0; otherwise, do not reject H_0.

Step 4 The t-statistic has df $= n - 1$. Use Table IV to estimate the P-value, or obtain it exactly by using technology.



Step 5 If $P \leq \alpha$, reject H_0; otherwise, do not reject H_0.

Step 6 Interpret the results of the hypothesis test.

Note: The hypothesis test is exact for normal populations and is approximately correct for large samples from nonnormal populations.

Section 9.6

The Wilcoxon Signed-Rank Test

Table 9.13

Sample of weekly food costs ($)

143	169	149	135	161
138	152	150	141	159

Table 9.14

Steps for ranking the data in Table 9.13 according to distance and direction from the null hypothesis mean

Cost ($) x	Difference $D = x - 157$	$\|D\|$	Rank of $\|D\|$	Signed rank R
143	−14	14	7	−7
138	−19	19	9	−9
169	12	12	6	6
152	−5	5	3	−3
149	−8	8	5	−5
150	−7	7	4	−4
135	−22	22	10	−10
141	−16	16	8	−8
161	4	4	2	2
159	2	2	1	1

Step 1 *Subtract μ_0 from x.*

Step 2 *Make each difference positive by taking absolute values.*

Step 3 *Rank the absolute differences in order from smallest (1) to largest (10).*

Step 4 *Give each rank the same sign as the sign in the Difference column.*

Procedure 9.3

Wilcoxon Signed-Rank Test

Purpose To perform a hypothesis test for a population mean, μ

Assumptions

1. Simple random sample
2. Symmetric population

Step 1 **The null hypothesis is H_0: $\mu = \mu_0$, and the alternative hypothesis is**

$$H_a: \mu \neq \mu_0 \text{ (Two tailed)} \quad \text{or} \quad H_a: \mu < \mu_0 \text{ (Left tailed)} \quad \text{or} \quad H_a: \mu > \mu_0 \text{ (Right tailed)}$$

Step 2 **Decide on the significance level, α.**

Step 3 **Compute the value of the test statistic**

$$W = \text{sum of the positive ranks}$$

and denote that value W_0. To do so, construct a work table of the following form.

Observation x	Difference $D = x - \mu_0$	$\|D\|$	Rank of $\|D\|$	Signed rank R
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.

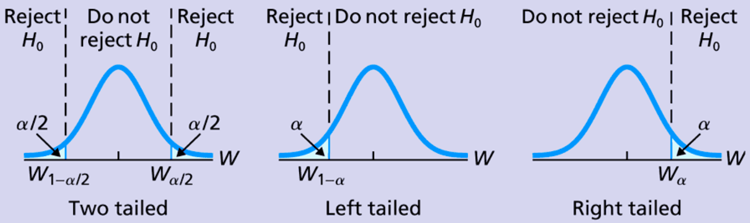
Procedure 9.3 (cont.)

CRITICAL-VALUE APPROACH	OR	*P*-VALUE APPROACH

Step 4 The critical value(s) are

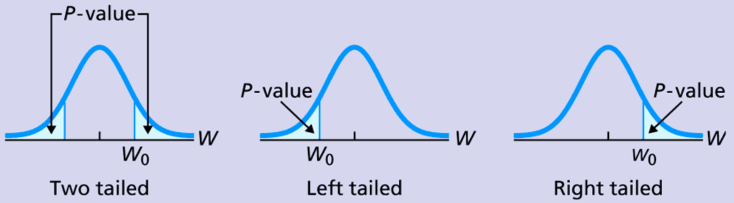
$W_{1-\alpha/2}$ and $W_{\alpha/2}$ (Two tailed) or $W_{1-\alpha}$ (Left tailed) or W_{α} (Right tailed)

Use Table V to find the critical value(s). For a left-tailed or two-tailed test, you will also need the relation $W_{1-A} = n(n+1)/2 - W_A$.



Step 5 If the value of the test statistic falls in the rejection region, reject H_0; otherwise, do not reject H_0.

Step 4 Obtain the P-value by using technology.



Step 5 If $P \leq \alpha$, reject H_0; otherwise, do not reject H_0.

Step 6 Interpret the results of the hypothesis test.

Section 9.7

Type II Error Probabilities; Power

Figure 9.28

Decision criterion for the gas mileage illustration
(α = 0.05, n = 30)

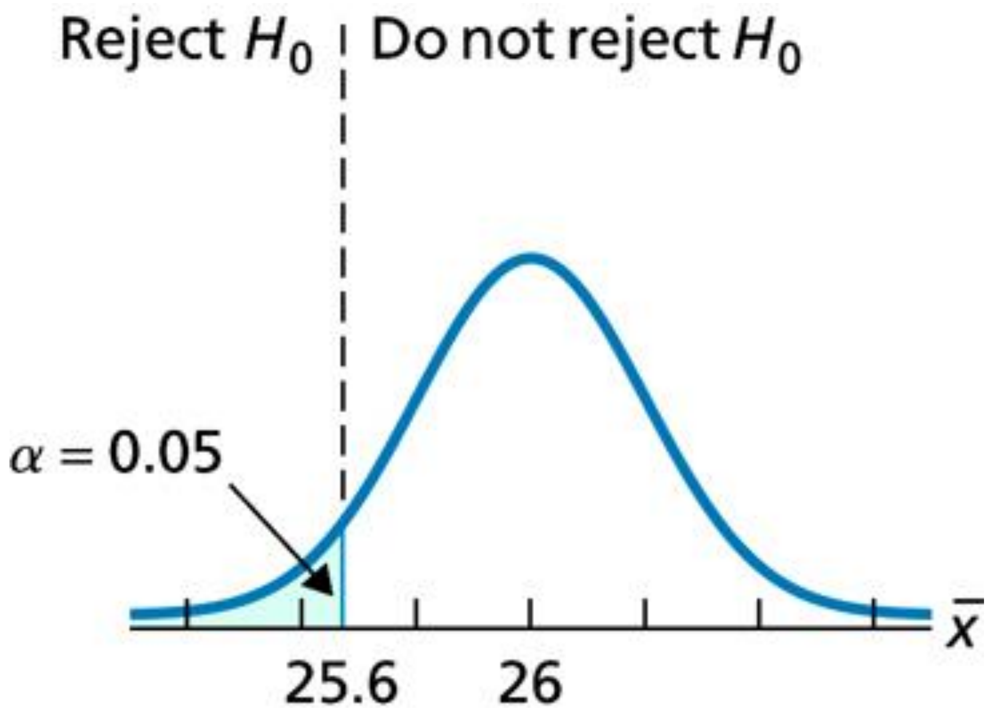


Figure 9.29

Determining the probability of a Type II error if   = 25.8 mpg

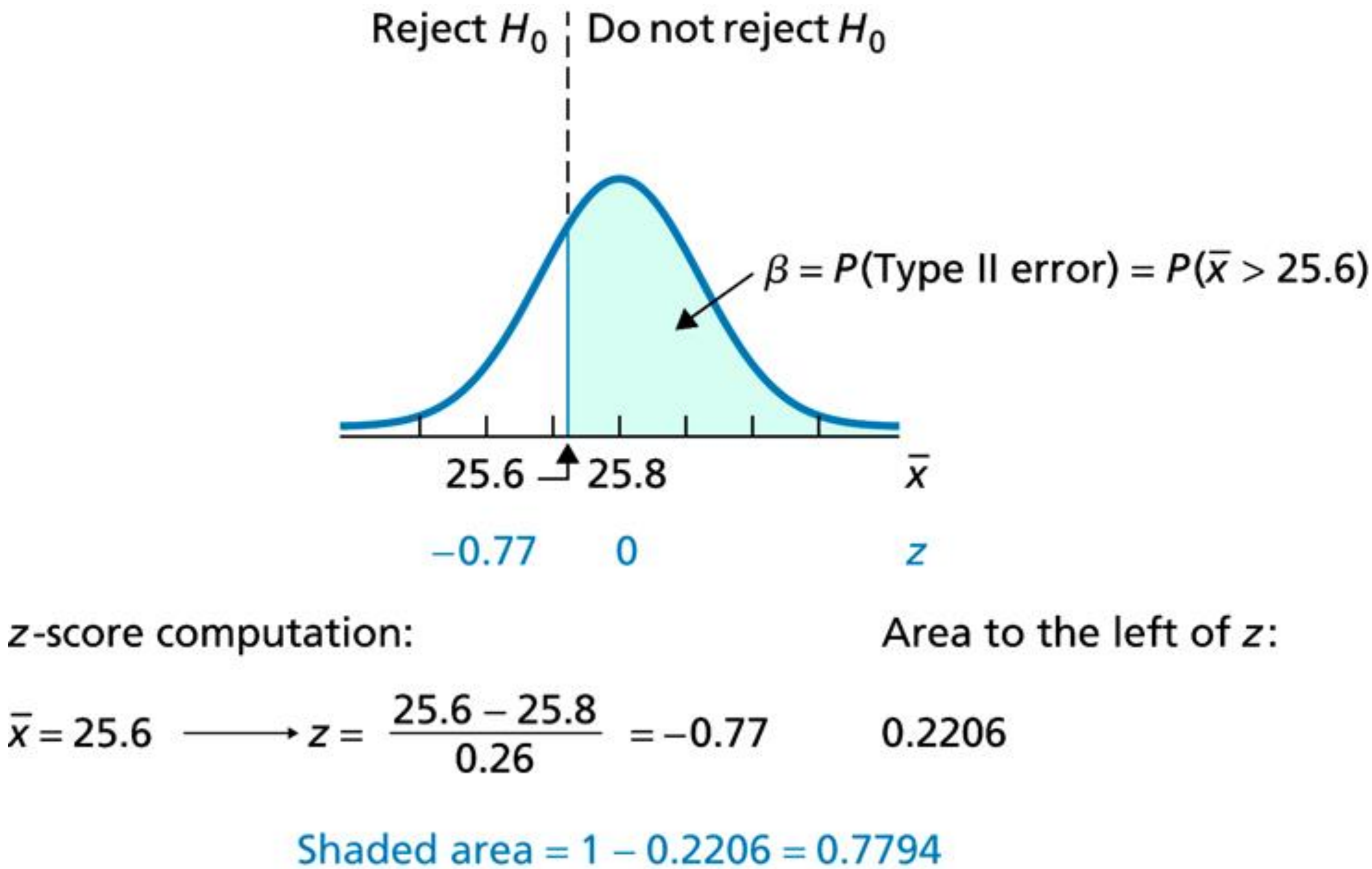
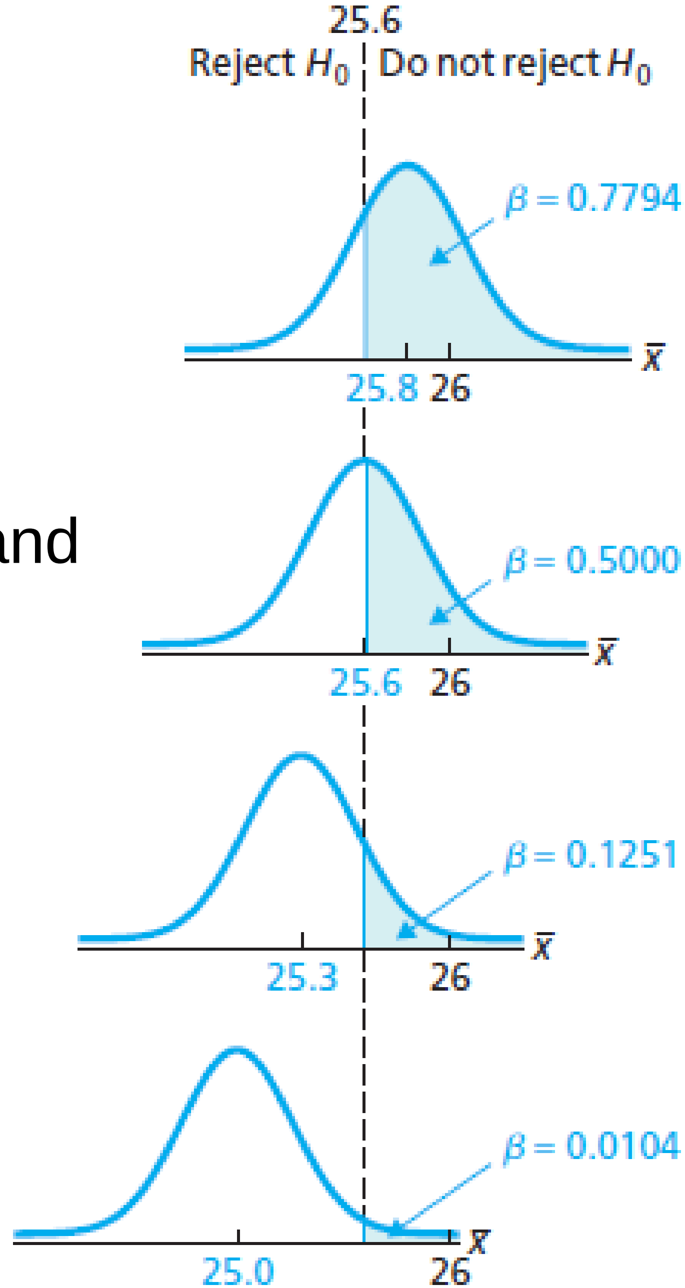


Figure 9.31

Type II error probabilities for = 25.8, 25.6, 25.3, and 25.0 (= 0.05, = 30)



Definition 9.6

Power

The **power** of a hypothesis test is the probability of not making a Type II error, that is, the probability of rejecting a false null hypothesis. We have

Power = (Type II error) = .

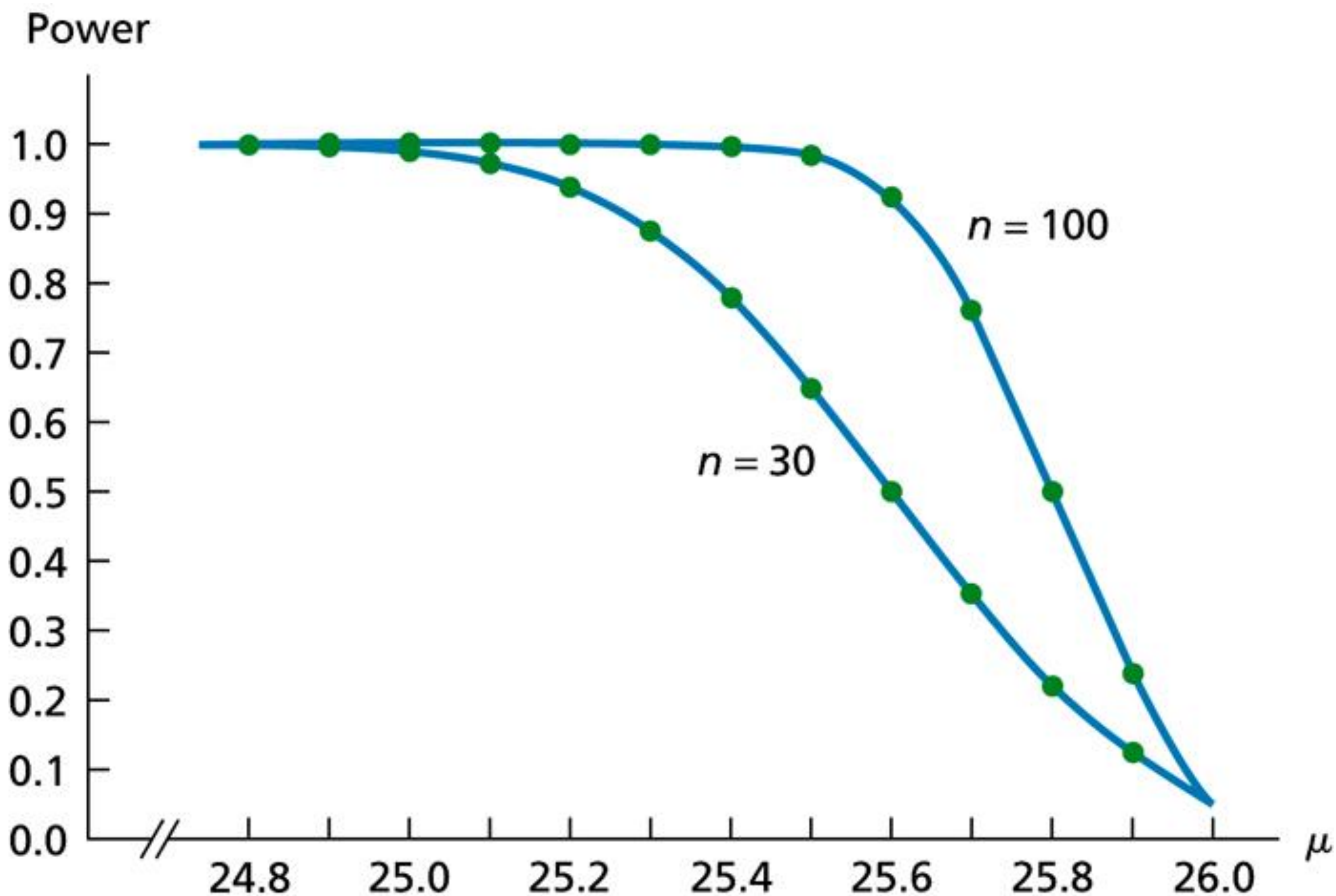
Table 9.16

Selected Type II error probabilities and powers for the gas mileage illustration
(= 0.05, = 30)

True mean μ	P (Type II error) β	Power $1-\beta$
25.9	0.8749	0.1251
25.8	0.7794	0.2206
25.7	0.6480	0.3520
25.6	0.5000	0.5000
25.5	0.3520	0.6480
25.4	0.2206	0.7794
25.3	0.1251	0.8749
25.2	0.0618	0.9382
25.1	0.0274	0.9726
25.0	0.0104	0.9896
24.9	0.0036	0.9964
24.8	0.0010	0.9990

Figure 9.34

Power curves for the gas mileage illustration when = 30 and = 100 (= 0.05)



Section 9.8

Which Procedure Should Be Used?

Figure 9.35

Flowchart for choosing the correct hypothesis testing procedure for a population mean