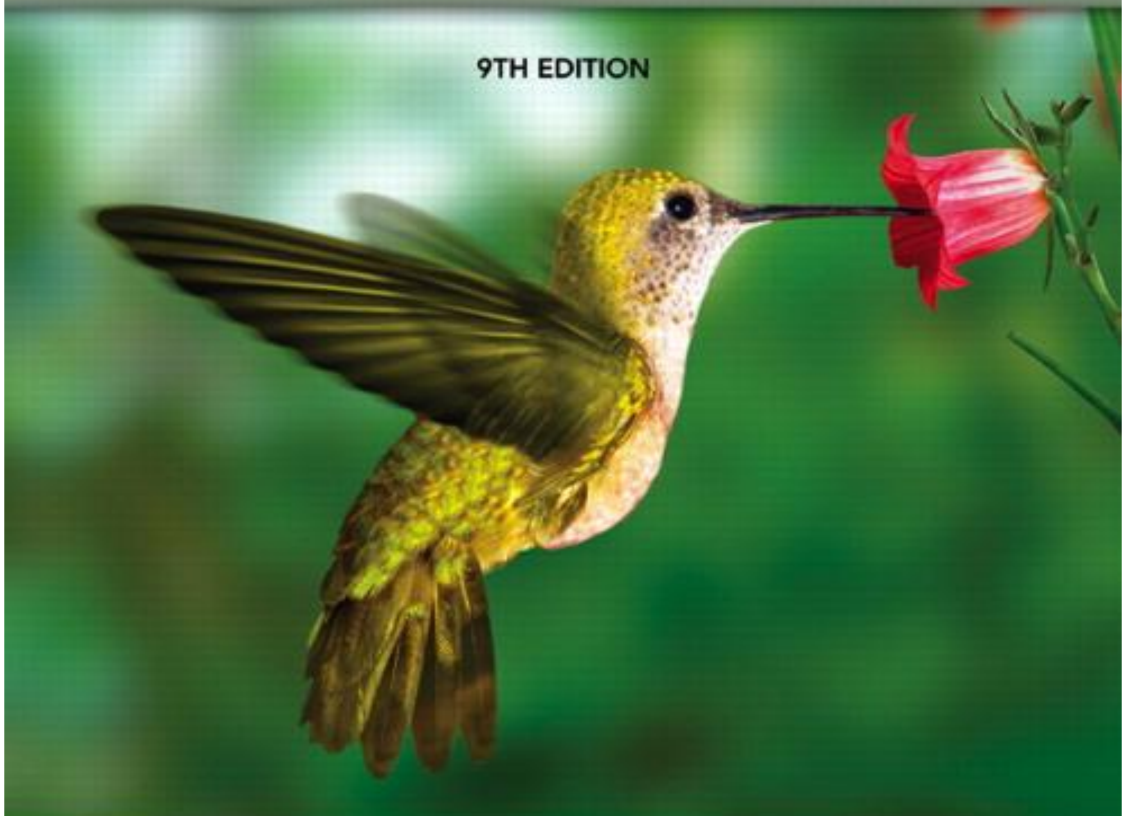
Introductory STATISTICS

9TH EDITION



Neil WEISS

Chapter 10

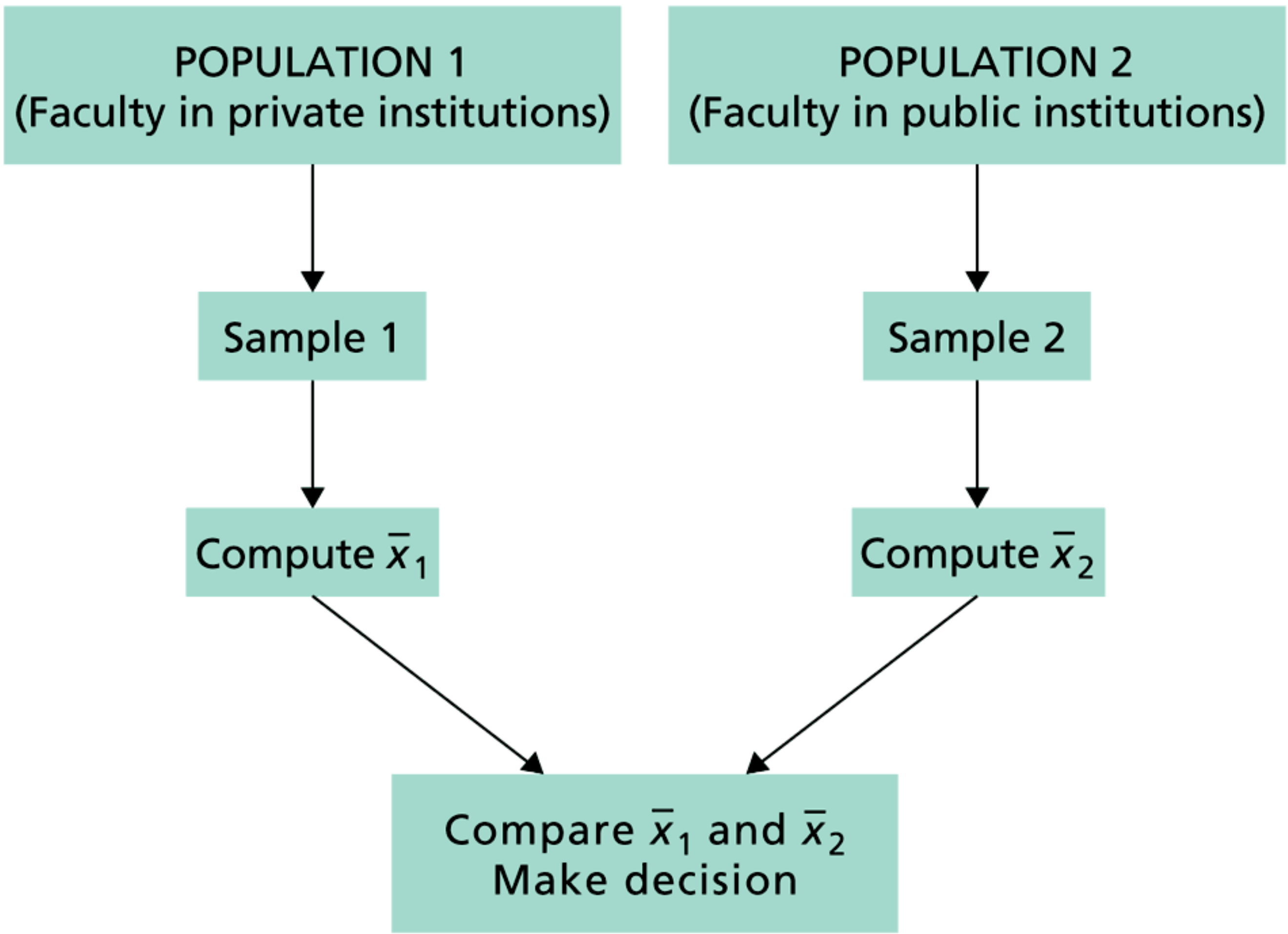
Inferences for Two Population Means

Section 10.1

The Sampling Distribution of the Difference between Two Sample Means for Independent Samples

Figure 10.1

Process for comparing two population means, using independent samples



Section 10.2

Inferences for Two Population Means, Using Independent Samples: Standard Deviations Assumed Equal

Procedure 10.1

Pooled t-Test

Purpose To perform a hypothesis test to compare two population means, μ_1 and μ_2

Assumptions

1. Simple random samples
2. Independent samples
3. Normal populations or large samples
4. Equal population standard deviations

Step 1 **The null hypothesis is H_0: $\mu_1 = \mu_2$, and the alternative hypothesis is**

H_a: $\mu_1 \neq \mu_2$		H_a: $\mu_1 < \mu_2$		H_a: $\mu_1 > \mu_2$
(Two tailed)	**or**	**(Left tailed)**	**or**	**(Right tailed)**

Step 2 **Decide on the significance level, α.**

Step 3 **Compute the value of the test statistic**

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s_p\sqrt{(1/n_1) + (1/n_2)}},$$

where

$$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}.$$

Denote the value of the test statistic t_0.

Procedure 10.1 (cont.)

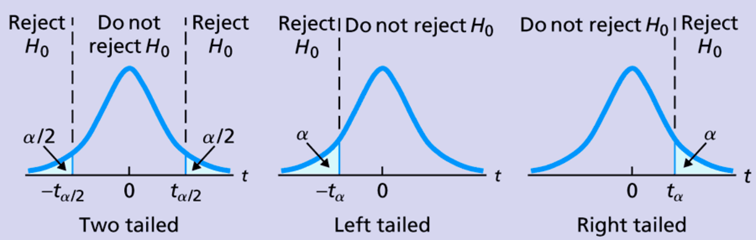
CRITICAL-VALUE APPROACH OR ***P*-VALUE APPROACH**

Critical-value approach

Step 4 The critical value(s) are

$\pm t_{\alpha/2}$ (Two tailed) or $-t_{\alpha}$ (Left tailed) or t_{α} (Right tailed)

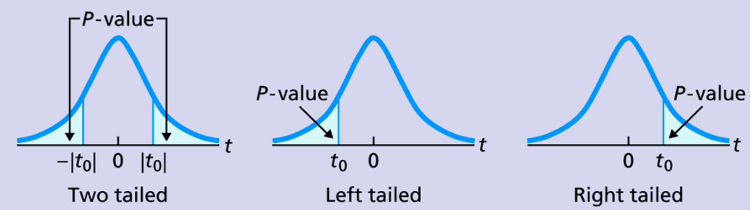
with df $= n_1 + n_2 - 2$. Use Table IV to find the critical value(s).



Step 5 If the value of the test statistic falls in the rejection region, reject H_0; otherwise, do not reject H_0.

P-value approach

Step 4 The t-statistic has df $= n_1 + n_2 - 2$. Use Table IV to estimate the P-value, or obtain it exactly by using technology.



Step 5 If $P \leq \alpha$, reject H_0; otherwise, do not reject H_0.

Step 6 Interpret the results of the hypothesis test.

Note: The hypothesis test is exact for normal populations and is approximately correct for large samples from nonnormal populations.

Procedure 10.2

Pooled *t*-Interval Procedure

Purpose To find a confidence interval for the difference between two population means, μ_1 and μ_2

Assumptions

1. Simple random samples
2. Independent samples
3. Normal populations or large samples
4. Equal population standard deviations

Step 1 **For a confidence level of $1 - \alpha$, use Table IV to find $t_{\alpha/2}$ with df $= n_1 + n_2 - 2$.**

Step 2 **The endpoints of the confidence interval for $\mu_1 - \mu_2$ are**

$$(\bar{x}_1 - \bar{x}_2) \pm t_{\alpha/2} \cdot s_p \sqrt{(1/n_1) + (1/n_2)}.$$

Step 3 **Interpret the confidence interval.**

Note: The confidence interval is exact for normal populations and is approximately correct for large samples from nonnormal populations.

Section 10.3

Inferences for Two Population Means, Using Independent Samples: Standard Deviations Not Assumed Equal

Procedure 10.3

Nonpooled *t*-Test

Purpose To perform a hypothesis test to compare two population means, μ_1 and μ_2

Assumptions

1. Simple random samples
2. Independent samples
3. Normal populations or large samples

Step 1 **The null hypothesis is H_0: $\mu_1 = \mu_2$, and the alternative hypothesis is**

H_a: $\mu_1 \neq \mu_2$ **(Two tailed)**	**or**	H_a: $\mu_1 < \mu_2$ **(Left tailed)**	**or**	H_a: $\mu_1 > \mu_2$ **(Right tailed)**

Step 2 **Decide on the significance level, α.**

Step 3 **Compute the value of the test statistic**

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{(s_1^2/n_1) + (s_2^2/n_2)}}.$$

Denote the value of the test statistic t_0.

Procedure 10.3 (cont.)

CRITICAL-VALUE APPROACH	OR	*P*-VALUE APPROACH

CRITICAL-VALUE APPROACH

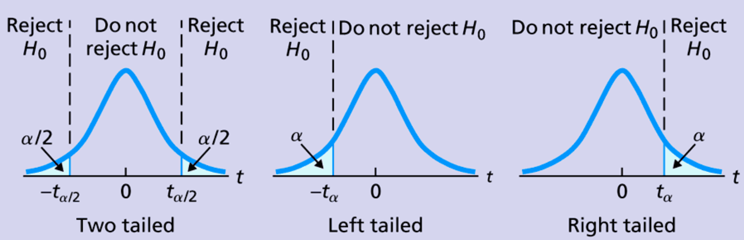
Step 4 The critical value(s) are

$\pm t_{\alpha/2}$ (Two tailed) or $-t_\alpha$ (Left tailed) or t_α (Right tailed)

with df $= \Delta$, where

$$\Delta = \frac{\left[(s_1^2/n_1) + (s_2^2/n_2)\right]^2}{\dfrac{(s_1^2/n_1)^2}{n_1 - 1} + \dfrac{(s_2^2/n_2)^2}{n_2 - 1}}$$

rounded down to the nearest integer. Use Table IV to find the critical value(s).



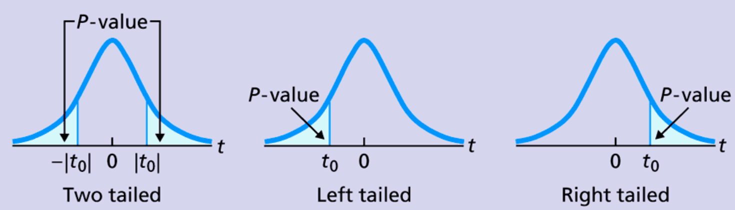
Step 5 If the value of the test statistic falls in the rejection region, reject H_0; otherwise, do not reject H_0.

P-VALUE APPROACH

Step 4 The t-statistic has df $= \Delta$, where

$$\Delta = \frac{\left[(s_1^2/n_1) + (s_2^2/n_2)\right]^2}{\dfrac{(s_1^2/n_1)^2}{n_1 - 1} + \dfrac{(s_2^2/n_2)^2}{n_2 - 1}}$$

rounded down to the nearest integer. Use Table IV to estimate the P-value, or obtain it exactly by using technology.



Step 5 If $P \le \alpha$, reject H_0; otherwise, do not reject H_0.

Step 6 Interpret the results of the hypothesis test.

Procedure 10.4

Nonpooled *t*-Interval Procedure

Purpose To find a confidence interval for the difference between two population means, μ_1 and μ_2

Assumptions

1. Simple random samples
2. Independent samples
3. Normal populations or large samples

Step 1 **For a confidence level of $1 - \alpha$, use Table IV to find $t_{\alpha/2}$ with df $= \Delta$, where**

$$\Delta = \frac{\left[(s_1^2/n_1) + (s_2^2/n_2)\right]^2}{\dfrac{(s_1^2/n_1)^2}{n_1 - 1} + \dfrac{(s_2^2/n_2)^2}{n_2 - 1}}$$

rounded down to the nearest integer.

Step 2 **The endpoints of the confidence interval for $\mu_1 - \mu_2$ are**

$$(\bar{x}_1 - \bar{x}_2) \pm t_{\alpha/2} \cdot \sqrt{(s_1^2/n_1) + (s_2^2/n_2)}.$$

Step 3 **Interpret the confidence interval.**

Section 10.4

The Mann-Whitney Test

Figure 10.9

Appropriate procedure for comparing two population means based on independent simple random samples

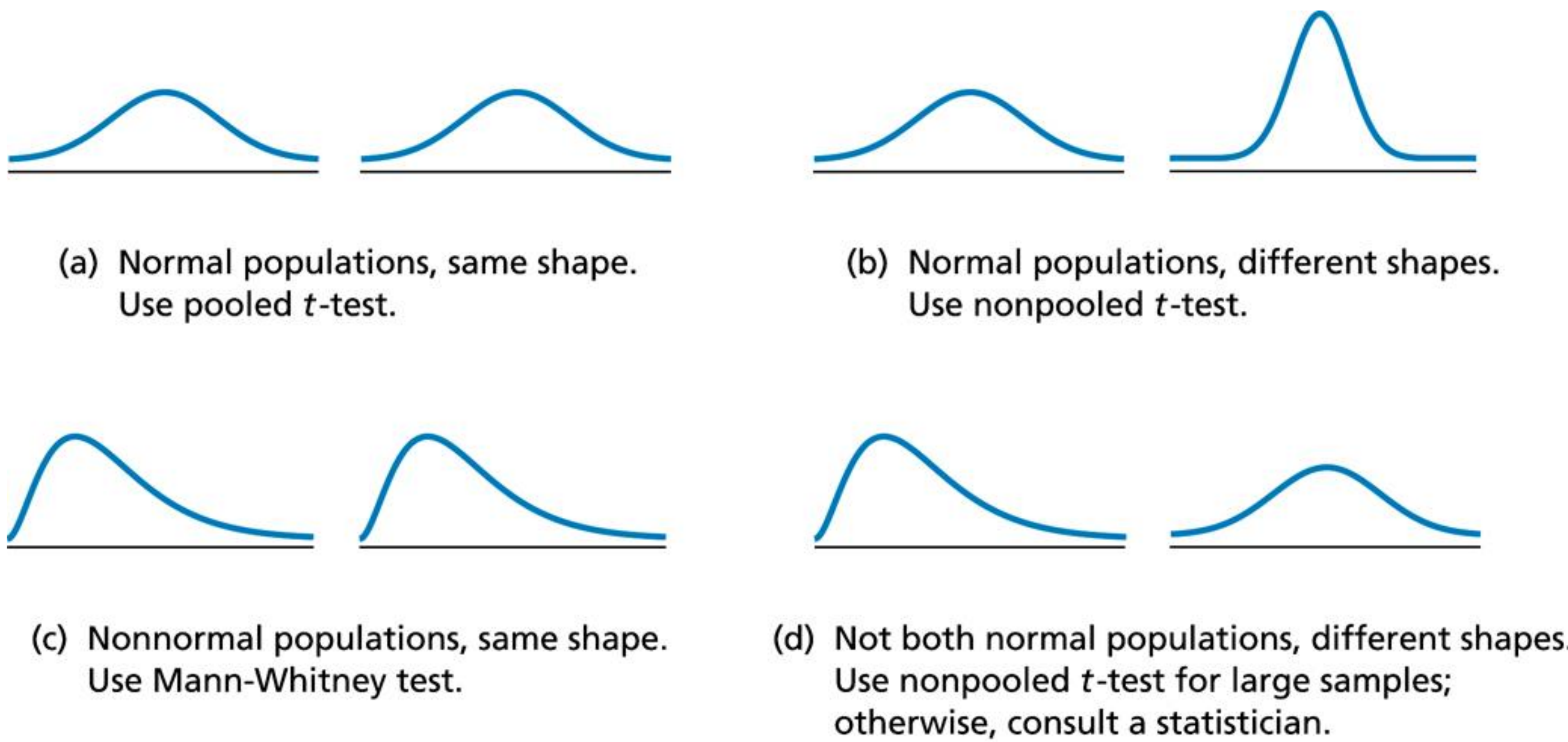


Table 10.9 & 10.10

Times, in minutes, required
to learn how to use the system

Without experience	With experience
139	142
118	109
164	130
151	107
182	155
140	88
134	95
	104

Results of ranking the
combined data from Table 10.9

Without experience	Overall rank	With experience	Overall rank
139	9	142	11
118	6	109	5
164	14	130	7
151	12	107	4
182	15	155	13
140	10	88	1
134	8	95	2
		104	3

Procedure 10.5

Mann–Whitney Test

Purpose To perform a hypothesis test to compare two population means, μ_1 and μ_2

Assumptions

1. Simple random samples
2. Independent samples
3. Same-shape populations

Step 1 **The null hypothesis is H_0: $\mu_1 = \mu_2$, and the alternative hypothesis is**

H_a: $\mu_1 \neq \mu_2$ (Two tailed) or H_a: $\mu_1 < \mu_2$ (Left tailed) or H_a: $\mu_1 > \mu_2$ (Right tailed)

Step 2 **Decide on the significance level, α.**

Step 3 **Compute the value of the test statistic**

$$M = \text{sum of the ranks for sample data from Population 1}$$

and denote that value M_0. To do so, construct a work table of the following form.

Sample from Population 1	Overall rank	Sample from Population 2	Overall rank
.	.	.	.
.	.	.	.
.	.	.	.

Procedure 10.5 (cont.)

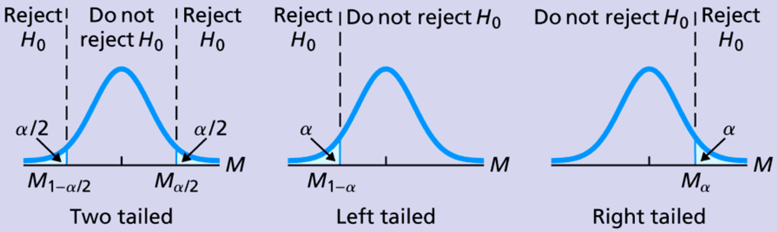
CRITICAL-VALUE APPROACH	OR	*P*-VALUE APPROACH

CRITICAL-VALUE APPROACH

Step 4 The critical value(s) are

$M_{1-\alpha/2}$ and $M_{\alpha/2}$ (Two tailed) or $M_{1-\alpha}$ (Left tailed) or M_{α} (Right tailed)

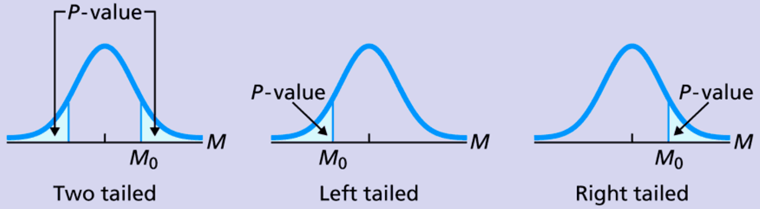
Use Table VI to find the critical value(s). For a left-tailed or two-tailed test, you will also need the relation $M_{1-A} = n_1(n_1 + n_2 + 1) - M_A$.



Step 5 If the value of the test statistic falls in the rejection region, reject H_0; otherwise, do not reject H_0.

***P*-VALUE APPROACH**

Step 4 Obtain the P-value by using technology.



Step 5 If $P \leq \alpha$, reject H_0; otherwise, do not reject H_0.

Step 6 Interpret the results of the hypothesis test.

Section 10.5

Inferences for Two Population Means, Using Paired Samples

Table 10.13

Ages, in years, of a random sample of 10 married couples

Couple	Husband	Wife	Difference, *d*
1	59	53	6
2	21	22	−1
3	33	36	−3
4	78	74	4
5	70	64	6
6	33	35	−2
7	68	67	1
8	32	28	4
9	54	41	13
10	52	44	8
			36

Procedure 10.6

Paired *t*-Test

Purpose To perform a hypothesis test to compare two population means, μ_1 and μ_2

Assumptions

1. Simple random paired sample
2. Normal differences or large sample

Step 1 The null hypothesis is H_0: $\mu_1 = \mu_2$, and the alternative hypothesis is

H_a: $\mu_1 \neq \mu_2$ (Two tailed) or H_a: $\mu_1 < \mu_2$ (Left tailed) or H_a: $\mu_1 > \mu_2$ (Right tailed)

Step 2 Decide on the significance level, α.

Step 3 Compute the value of the test statistic

$$t = \frac{\bar{d}}{s_d/\sqrt{n}}$$

and denote that value t_0.

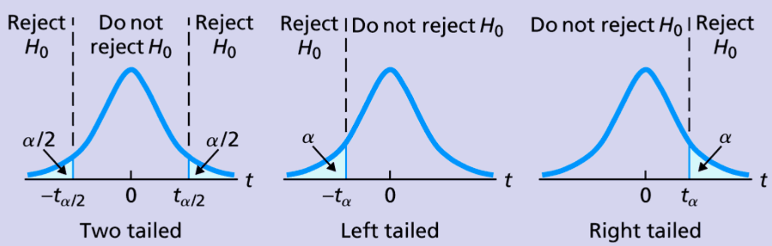
Procedure 10.6 (cont.)

CRITICAL-VALUE APPROACH OR ***P*-VALUE APPROACH**

Step 4 The critical value(s) are

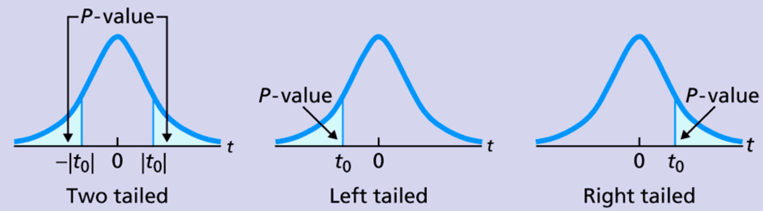
$\pm t_{\alpha/2}$ (Two tailed) or $-t_{\alpha}$ (Left tailed) or t_{α} (Right tailed)

with df $= n - 1$. Use Table IV to find the critical value(s).



Step 5 If the value of the test statistic falls in the rejection region, reject H_0; otherwise, do not reject H_0.

Step 4 The t-statistic has df $= n - 1$. Use Table IV to estimate the P-value, or obtain it exactly by using technology.



Step 5 If $P \leq \alpha$, reject H_0; otherwise, do not reject H_0.

Step 6 Interpret the results of the hypothesis test.

Note: The hypothesis test is exact for normal differences and is approximately correct for large samples and nonnormal differences.

Procedure 10.7

Paired t-Interval Procedure

Purpose To find a confidence interval for the difference between two population means, μ_1 and μ_2

Assumptions

1. Simple random paired sample

2. Normal differences or large sample

Step 1 **For a confidence level of $1 - \alpha$, use Table IV to find $t_{\alpha/2}$ with $\text{df} = n - 1$.**

Step 2 **The endpoints of the confidence interval for $\mu_1 - \mu_2$ are**

$$\bar{d} \pm t_{\alpha/2} \cdot \frac{s_d}{\sqrt{n}}.$$

Step 3 **Interpret the confidence interval.**

Note: The confidence interval is exact for normal differences and is approximately correct for large samples and nonnormal differences.

Section 10.6

The Paired Wilcoxon Signed-Rank Test

Procedure 10.8

Paired Wilcoxon Signed-Rank Test

Purpose To perform a hypothesis test to compare two population means, μ_1 and μ_2

Assumptions

1. Simple random paired sample
2. Symmetric differences

Step 1 **The null hypothesis is H_0: $\mu_1 = \mu_2$, and the alternative hypothesis is**

H_a: $\mu_1 \neq \mu_2$ (Two tailed)	or	H_a: $\mu_1 < \mu_2$ (Left tailed)	or	H_a: $\mu_1 > \mu_2$ (Right tailed)

Step 2 **Decide on the significance level, α.**

Step 3 **Compute the value of the test statistic**

$$W = \text{sum of the positive ranks}$$

and denote that value W_0. To do so, first calculate the paired differences of the sample pairs, next discard all paired differences that equal 0 and reduce the sample size accordingly, and then construct a work table of the following form.

Paired difference d	$\lvert d \rvert$	Rank of $\lvert d \rvert$	Signed rank R
.	.	.	.
.	.	.	.
.	.	.	.

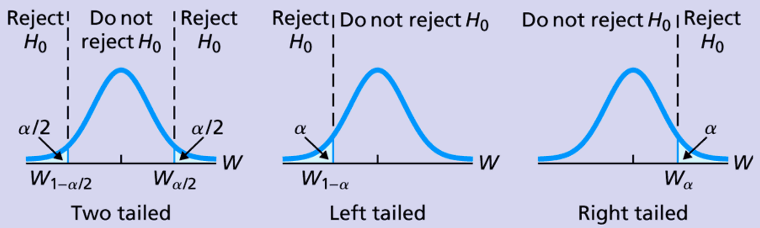
Procedure 10.8 (cont.)

CRITICAL-VALUE APPROACH	OR	*P*-VALUE APPROACH

Step 4 The critical value(s) are

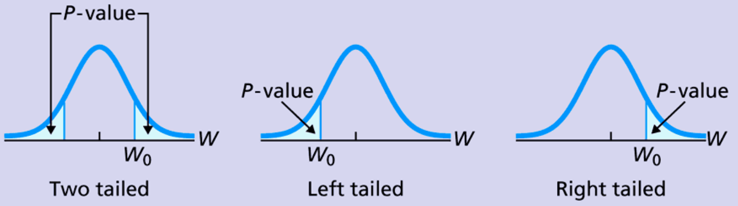
$W_{1-\alpha/2}$ and $W_{\alpha/2}$ (Two tailed) or $W_{1-\alpha}$ (Left tailed) or W_{α} (Right tailed)

Use Table V to find the critical value(s). For a left-tailed or two-tailed test, you will also need the relation $W_{1-A} = n(n+1)/2 - W_A$.



Step 5 If the value of the test statistic falls in the rejection region, reject H_0; otherwise, do not reject H_0.

Step 4 Obtain the *P*-value by using technology.



Step 5 If $P \leq \alpha$, reject H_0; otherwise, do not reject H_0.

Step 6 Interpret the results of the hypothesis test.

Section 10.7
Which Procedure Should Be Used?

Table 10.15

Summary of hypothesis-testing procedures for comparing two population means. The null hypothesis for all tests is

Type	Assumptions	Test statistic	Procedure to use
Pooled t-test	1. Simple random samples 2. Independent samples 3. Normal populations or large samples 4. Equal population standard deviations	$t = \dfrac{\bar{x}_1 - \bar{x}_2}{s_p\sqrt{(1/n_1) + (1/n_2)}}$ † $(\text{df} = n_1 + n_2 - 2)$	10.1 (page 441)
Nonpooled t-test	1. Simple random samples 2. Independent samples 3. Normal populations or large samples	$t = \dfrac{\bar{x}_1 - \bar{x}_2}{\sqrt{(s_1^2/n_1) + (s_2^2/n_2)}}$ ‡	10.3 (page 453)
Mann–Whitney test	1. Simple random samples 2. Independent samples 3. Same-shape populations	M = sum of the ranks for sample data from Population 1	10.5 (page 468)
Paired t-test	1. Simple random paired sample 2. Normal differences or large sample	$t = \dfrac{\bar{d}}{s_d/\sqrt{n}}$ $(\text{df} = n - 1)$	10.6 (page 481)
Paired W-test	1. Simple random paired sample 2. Symmetric differences	W = sum of positive ranks	10.8 (page 492)

† $s_p = \sqrt{\dfrac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$ ‡ $\text{df} = \dfrac{[(s_1^2/n_1) + (s_2^2/n_2)]^2}{\dfrac{(s_1^2/n_1)^2}{n_1 - 1} + \dfrac{(s_2^2/n_2)^2}{n_2 - 1}}$

Figure 10.19

Flowchart for choosing the correct hypothesis-testing procedure for comparing two population means

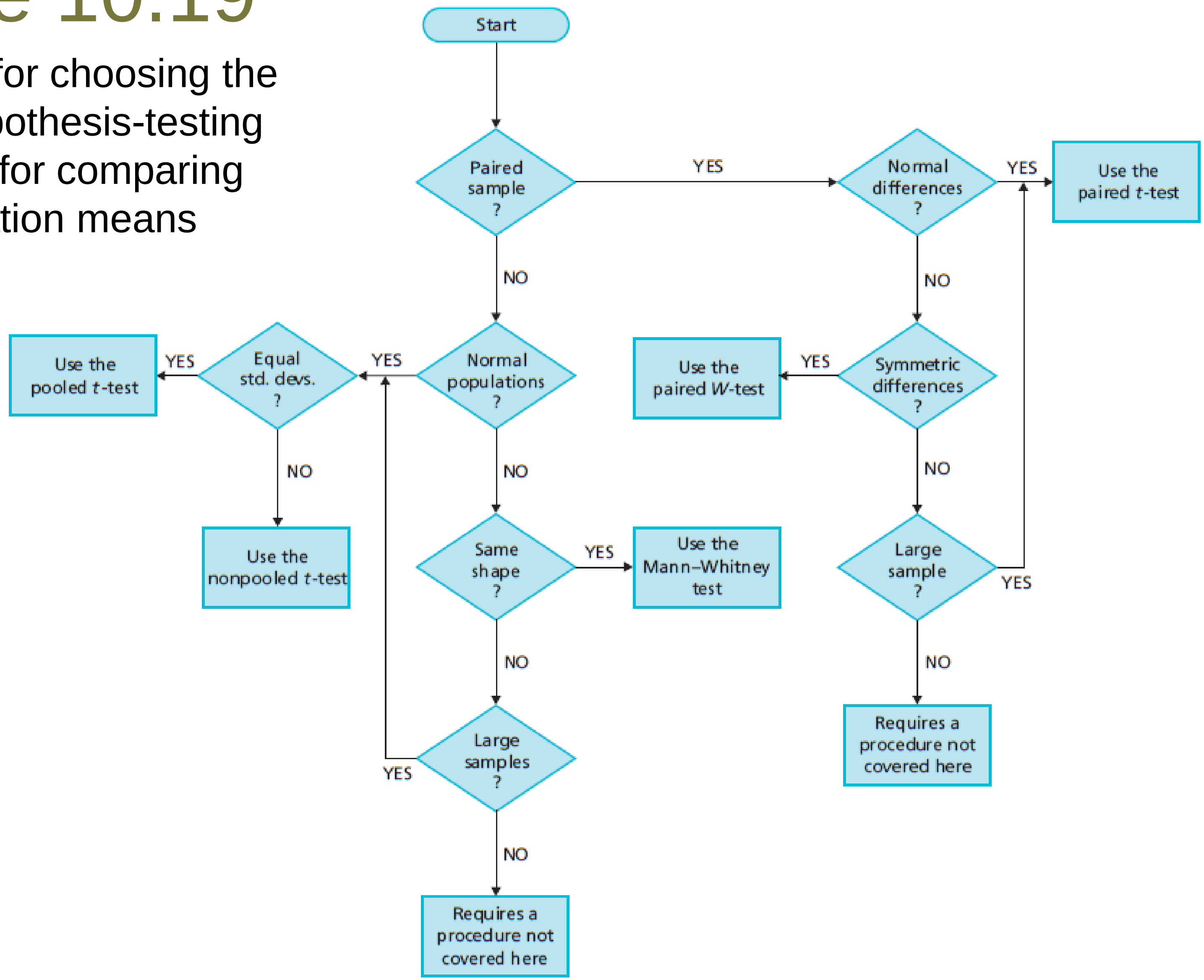
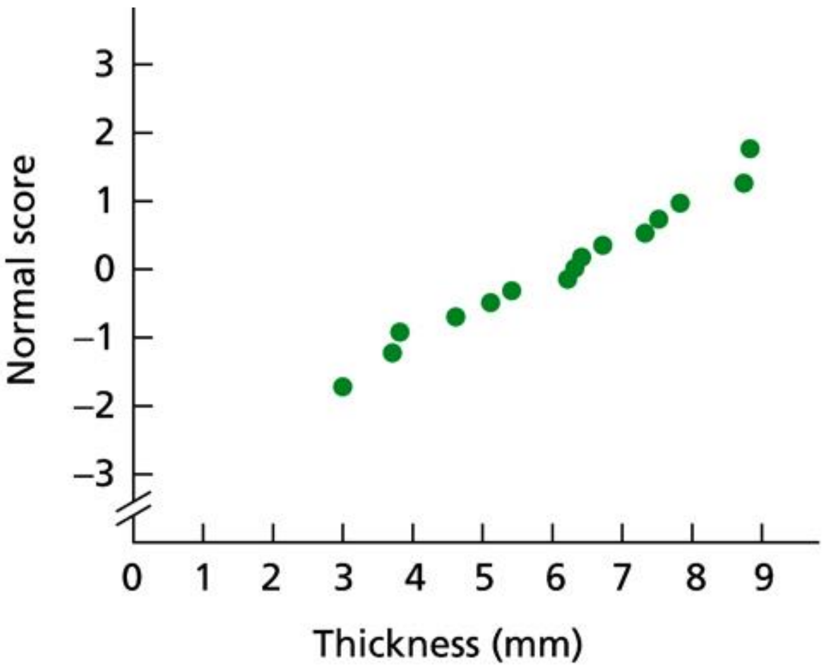
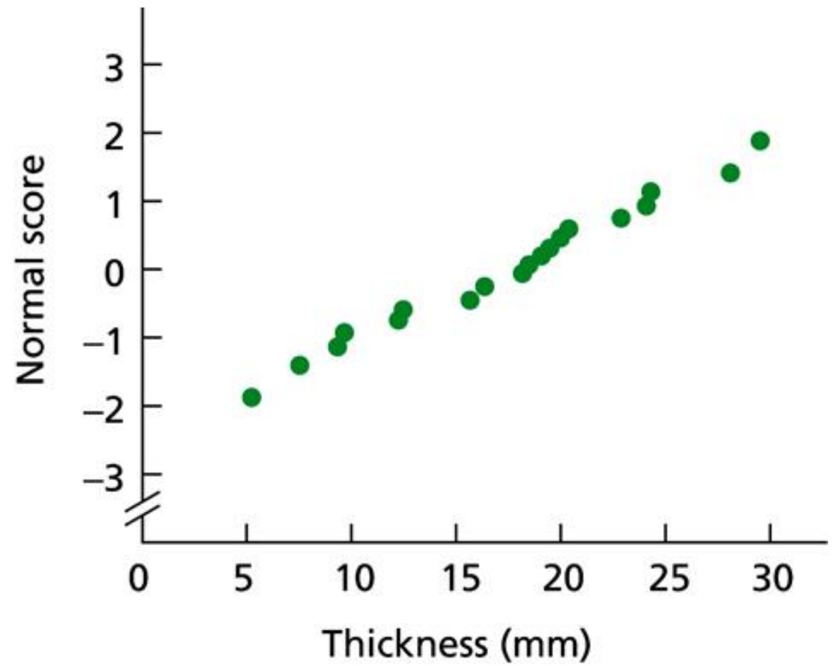


Figure 10.20

Normal probability plots of the sample data for (a) elite runners and (b) others



(a) Runners



(b) Others

Figure 10.21

Boxplots of the sample data for elite runners and others